Using learners' responses to inform the teaching of mathematics

Resource materials based on the Annual National Assessments

Grade 3

basic education
Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

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Foreword from the Minister of Basic Education

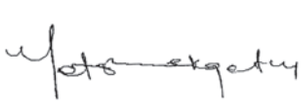
It is my pleasure to present this set of materials to be part of the range of resources made available for our teachers to improve the quality of their teaching. Since 2011, the Department of Basic Education (DBE) has been conducting the Annual National Assessment (ANA) on Grades 1-6 and 9 learners in Language and Mathematics.

The diagnostic reports produced after the administration of the ANA point to areas where individual teachers need specific support in terms of effective methods of facilitating learning. One of these areas is the utilisation of assessment data in a manner which will inform improved teaching.

This set of materials has therefore been developed to build the capacity of teachers to analyse the ANA and other tests in order to identify typical errors made by learners and thereafter select appropriate teaching strategies to correct these errors and improve the teaching of Mathematics. The materials give precise and useful guidelines, with regards to accurate identification of the challenging content and conceptual areas shown through the errors as well as using these errors to create opportunities for learners to improve their mathematical abilities.

I wish to express my sincere gratitude to our partners, the United Nations International Children's Emergency Fund (UNICEF) and JET Education Services for their invaluable contribution in making this resource available.

I am confident that teachers will find the materials useful and that this intervention will make a meaningful contribution to their teaching and professional development.



MRS ANGIE MOTSHEKGA, MP
MINISTER OF BASIC EDUCATION

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General sections

Introduction

All South African learners in Grades 1 to 6 and 9 annually write the Annual National Assessments, which have become known colloquially as 'the ANAs'. The Department of Basic Education's diagnostic reports on the ANA results identified a need to strengthen the formative use of assessment data to support teaching. This resource book has been created to meet this need.

The material presented here focuses on mathematics topics covered in both the 2013 and 2014 ANAs and deals with the errors made by learners as shown in their responses to the ANA questions. In addition to highlighting learners' misconceptions, analysis of the learners' responses also reveals the correct understanding of the topics tested.

Following the error analysis of responses in the ANAs, teaching strategies to address these mistakes and misconceptions are presented. Both the error analysis and the teaching strategies in this resource book can be applied to other assessment questions that test the same topic, or used in the normal course of teaching the topic.

The materials presented here are, however, not meant to be exhaustive in terms of content covered in the mathematics curriculum, but are rather a living and growing resource that can be added to in response to future ANAs. Teachers are encouraged to add their own examples of learners' responses to assessment questions as well as to the teaching strategies which address the misconceptions noted in the learners' responses.

The following section of this book explains the concept of error analysis and how to conduct an analysis of learner's responses in an assessment or test. Next, guidelines on how to use these materials in conjunction with error analysis are presented. Finally, actual error analysis of learners' responses in the ANAs and ideas for teaching strategies to address the identified errors and misconceptions are presented per topic.

What is error analysis?

The formative use of assessment data to support teaching can be achieved by what researchers call “error analysis”.

Error analysis, also referred to as error pattern analysis, is a multifaceted activity involving the study of errors in learners' work with a view to finding explanations for learners' reasoning errors.

It is important to note that not all errors are reasoning faults; some are simply careless errors which educational researchers have termed “slips”. Slips can easily be corrected if the faulty process is pointed out to the learner. Slips are random errors that learners may make (e.g. reversing a number) and do not indicate systematic misconceptions or conceptual problems.

Error analysis is concerned with identifying and addressing the common errors (or 'bugs') which learners make due to their lack of conceptual or procedural understanding. These types of mathematical errors occur when the learner believes that what has been done is correct – showing that the learner's reasoning is faulty. Researchers have termed these errors systematic and persistent errors. Unless the misconceptions that cause the learners to make these errors are corrected, learners will repeat them over and over again and not be aware that they are using incorrect procedures to solve problems. These systematic errors can be said to be the result of the use of algorithms that lead to incorrect answers or the use of procedures that have not been fully understood.

Error analysis is important as it does not just mean analysing learners' steps in finding the solutions to a problem, but also involves finding the best ways to remediate the misconceptions the analysis shows. Rectifying learners' misconceptions enables teachers to make sure that learners can move on to the next step in the curriculum with a sound base.

Many writers on education have pointed out that the ability of teachers to understand and remediate common learner errors and misconceptions is an important part of what teachers should know, that is, of the pedagogical content knowledge that teachers should have. In order to conduct error analysis efficiently, teachers need to have a good knowledge of mathematical content, a good grasp of their learners' levels of mathematical understanding and a well-grounded understanding of the learner and how a learner learns.

Shulman (1986)[1] was a writer who developed a theory of teacher knowledge. He

[1] Shulman, L.S. 1986. Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2):4–14.

included teachers' knowledge of learners' levels of understanding as an important part of teacher knowledge and explained that this knowledge helps teachers to become aware of the process of learning mathematics as well as to understand the mathematical concepts that learners struggle to grasp. Other authors describe how error analysis and efforts to understand learners' levels of mathematical understanding build teacher's own knowledge of the underlying cognitive processes involved.

In conclusion, error analysis is a valuable activity that helps teachers to understand some of the thinking of their learners. Understanding the way learners think can assist teachers to adjust their teaching strategies and classroom and assessment practices and may ultimately lead to improvement in learner achievement.

How to analyse your learners' responses to assessment questions?

This section contains a fictional ten-question test which is used to illustrate how to design and use an assessment grid to analyse your class's results in a test or assessment. This is done in a step-by-step fashion.

The assessment grid, as the example given shows, reflects:

- The content addressed in each question;
- The question number;
- The marks allocated;
- The total of correct answers for each learner;
- The percentage each learner achieved;
- The learners' names; and
- The average score per question.

Step 1: Drawing up an assessment grid

- The first step in drawing up the grid is to identify the content addressed in each question in the assessment. The descriptions of this content make up the first heading row of the grid. It helps to be as specific as possible when identifying the content or skill being tested, as in the descriptions in blue in the example.
- You also need to record the marks (see red) allocated for each question and the total possible marks for the test (see green).

Content addressed in question	Simplification of expressions involving division	Equation involving exponents	Factorisation of expression	n^{th} term of a sequence	Simple interest	Compound interest	Congruent triangles	Drawing the reflected image	Surface area and volume of triangular prism	Determining gradient of line	**Total correct**	**Percentage**
Question number	1	2	3	4	5	6	7	8	9	10		
Mark	3	2	2	4	5	4	1	2	6	1	30	
Learners' name												
Average per question												

Step 2: Recording the results of each learner

In the next step you have to record the results per learner per question.

- First list all the learners in your class. We have 5 learners in our fictional class (see red).
- We have recorded the mark each learner got for each question (see blue).
- We also recorded the total marks each learner got (see green) and each learner's percentage (see pink).
- You might want to an additional column for the level code.

- Looking at the grid you will start to notice individual differences, e.g. learners that got the same total mark (see purple) did not get the same questions right or wrong.
- Also, you can identify which learners are doing well and which need additional support, e.g. Rosi and Lizzy are doing well, but Themba needs additional support.

Content addressed in question	Simplification of expressions involving division	Equation involving exponents	Factorisation of expression	n^{th} term of a sequence	Simple interest	Compound interest	Congruent triangles	Drawing the reflected image	Surface area and volume of triangular prism	Determining gradient of line	**Total correct**	**Percentage**
Question number	1	2	3	4	5	6	7	8	9	10		
Mark	3	2	2	4	5	4	1	2	6	1	30	%
Learners' name												
Rosi Thladi	2	1	2	2	3	3	1	1	4	1	20	67
Themba Hlambelo	1	1	1	0	2	2	1	0	2	0	10	33
Luneta Petersen	3	2	2	1	5	2	0	0	0	0	15	50
Johan De Wit	1	2	1	1	2	2	1	1	3	1	15	50
Lizzy Gregory	3	2	2	2	4	3	1	2	4	1	24	80
Average per question												

Step 3: Working out the class average

The third step is to work out and record the class average for each question.

- To calculate your class average, add up all the learners' averages and then divide by the number of learners in the class, e.g, Class average = 67 + 33 + 50 + 50 + 83 (see pink) ÷ 5 learners in class x 100 = 57%

Content addressed in question	Simplification of expressions involving division	Equation involving exponents	Factorisation of expression	n^{th} term of a sequence	Simple interest	Compound interest	Congruent triangles	Drawing the reflected image	Surface area and volume of triangular prism	Determining gradient of line	**Total correct**	**Percentage**
Question number	1	2	3	4	5	6	7	8	9	10		
Mark	3	2	2	4	5	4	1	2	6	1	30	%
Learners' name												
Rosi Thladi	2	1	2	2	3	3	1	1	4	1	20	67
Themba Hlambelo	1	1	1	0	2	2	1	0	2	0	10	33
Luneta Petersen	3	2	2	1	5	2	0	0	0	0	15	50
Johan De Wit	1	2	1	1	2	2	1	1	3	1	15	50
Lizzy Gregory	3	2	2	2	4	3	1	2	4	1	24	80
Average per question												60

Step 4: Working out the class average per question

In this step you want to determine the average your class got for each question. This will allow you to see the areas your learners are doing well in and the areas in which they need additional support.

- First work out the maximum possible marks the class could get for each question by multiplying the marks allocated to the question by the number of learners in the class, e.g., question 1 counts out of 3 marks, so the maximum possible total for the class would be 3 marks x 5 learners = 15 marks.
- Now add up all the learners' marks for question 1 (see red) to get the total marks achieved by the class for question 1, i.e. 2 + 1 + 3 + 1 + 3 = 10.
- To determine the class average for question 1 (see blue) divide the total marks achieved by the maximum possible marks and multiply the answer by 100, i.e. 10 ÷ 15 x 100 = 67% (see blue).

Content addressed in question	Simplification of expressions involving division	Equation involving exponents	Factorisation of expression	n^{th} term of a sequence	Simple interest	Compound interest	Congruent triangles	Drawing the reflected image	Surface area and volume of triangular prism	Determining gradient of line	**Total correct**	**Percentage**
Question number	1	2	3	4	5	6	7	8	9	10		
Mark	3	2	2	4	5	4	1	2	6	1	30	%
Learners' name												
Rosi Thladi	2	1	2	2	3	3	1	1	4	1	20	67
Themba Hlambelo	1	1	1	0	2	2	1	0	2	0	10	33
Luneta Petersen	3	2	2	1	5	2	0	0	0	0	15	50
Johan De Wit	1	2	1	1	2	2	1	1	3	1	15	50
Lizzy Gregory	3	2	2	2	4	3	1	2	4	1	24	80
Average per question	67	80	80	30	64	60	80	40	43	60		60

- Once you have done the same for all the questions, note which questions your class did well in (e.g. questions 2, 3 and 7) and which they did not do so well in (i.e. questions 8 and 9).
- Try to identify why this might be the case.

- Have you, for example, not spent enough time on these content areas?
- Do learners have a specific misunderstanding or misconception of this content area?
- Or are learners maybe making careless mistakes?

Step 5: Determine what learners should know to answer a specific question correctly

Let's consider what learners need to know to, for example, calculate the surface area of a triangular prism (question 9 in our fictional test). Learners should be able to:

- Deconstruct the prism into its net;
- Calculate the area of 2-D shapes, for example, triangles and rectangles;
- Write down the formulae for calculating the surface area of prisms;
- Substitute correctly within the formulae for calculating the surface area of prisms and cylinders.

You will note some of the content a learner needs to know in order to solve the problem correctly might have been covered in previous grades.

Step 6: Identify the typical errors learners make

Look at a selection of learners' responses and identify the typical errors they make. For example for question 9, you might note that learners:

- Neglected to calculate all the surface areas of the prism;
- Calculated the surface area of a specific face incorrectly; or
- Calculated all the surface areas of the prism correctly, but then added them incorrectly.

- It is useful here to have a discussion with specific learners or with the class to identify the reasoning learners used to solve the problem. By asking a learner to talk you through the process he/she followed to solve the problem, you could identify where the learner's logic went wrong or if the learner just made a careless calculation error.

Step 7: Determine appropriate teaching strategies to address learner errors and misconceptions

Once you have determined the reasoning or calculation errors learners made in a particular topic, you need to take steps to address these misconceptions or errors.

How to use these materials

The main section of this resource is organised according to topic in the following way:

- The topic and question used in the ANA to test it;
- The knowledge and skills required to answer the question and where the topic is located in the CAPS;
- Examples of learners' responses showing full, partial or no understanding of the question;
- Statistics showing the percentage of learners countrywide that answered the question correctly;
- Reasons the learners may have found it difficult to answer the question correctly;
- Teaching strategies to rectify the misconceptions that caused the errors; and (in some instances)
- Additional examples of how to test the topic.

To use the material

1. First determine which topic your class is experiencing difficulties with, then refer to the topic in the resource book.
   - Take note of what learners need to know to answer the question correctly and where the topic is found in the CAPS.
   - You should reflect critically on your teaching practice and the learners' knowledge with regard to the content and skills needed to answer the question. For instance, ask yourself:
     - Has sufficient time been spent on the required content and skills?
     - Have learners had sufficient time and practice to master the requisite skills?
   - You might need to revise the content and skills needed for a particular topic before attempting to remediate the learners' performance.
   - The content or skills might have been taught in a previous grade, but it is essential to make sure the learners have a solid foundation on which to build conceptual understanding of new topics.

2. Secondly, establish what kinds of slips and errors your learners make by either:
   - Looking at their answers to the ANA question (remember, you could give the learners the question in class to solve as part of class work); or
   - Looking at their answers to similar questions found in textbooks or class exercises.
   - You can refer to the examples of typical learner responses to assist you.

3. Take note of the statistics that follow the examples of learner responses.
   - The statistics will give you an indication of how difficult the questions should be for your class.

- If your class is doing better than the national sample, then it means you have started to lay a solid foundation and should continue your systematic teaching of the topic.

4. Lastly, look at and use the recommended teaching strategies to remedy the learners' errors and misconceptions as shown by the error analysis.
   - The teaching strategies are linked to the particular errors and misconceptions shown in this book – you may identify others made by your own learners.
   - The strategies are not meant as an exhaustive list of teaching strategies or exercises to do with learners on a specific topic, but as a starting point for remediation.
   - You will need to supplement the teaching strategies with your own ideas.

Note: Remember to teach **all the topics specified in the CAPS**, not just the ones featured in this resource.

Happy teaching!

Extra reading on error analysis

If you would like to know more about the theory of error analysis and how it can be used to aid you in your teaching, the following articles and books are suggested reading.

Allsopp, D.H., Kuger, M.H. & Lovitt, L.H. (2007). *Teaching mathematics meaningfully: Solutions for reaching struggling learners*. Baltimore: Paul H. Brooks.

Ashlock, R.B. (2006). *Error patterns in computation: Using error patterns to improve instruction. 9th edition*. New Jersey: Pearson.

Franke, M.L. & Kazemi, E. (2001). *Learning to teach mathematics: Focus on student thinking. Theory into Practice*, 40(2):102–109.

Herholdt, R & Sapire, I. (2014). An error analysis in early grades mathematics – A learning opportunity? *South African Journal of Childhood Education*, 4(1), 42-60.

Hill, H.C., Ball, D.L. & Schilling, S.C. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal of Research in Mathematics Education*, 39(4):372–400.

Ketterlin-Geller, L.R. & Yovanoff, P. (2009). Diagnostic assessments in mathematics to support instructional decision making. *Practical Assessment, Research &*

Evaluation, 14(16). Retrieved from http://pareonline.net/getvn.asp?v=14&n=16 (accessed on 19 April 2014).

McGuire, P. (2013). Using online error analysis items to support pre-service teachers' pedagogical content knowledge in mathematics. *Contemporary Issues in Technology and Teacher Education*, 13(3). Retrieved from http://www.citejournal.org/vol13/iss3/mathematics/article1.cfm (accessed on 19 April 2014).

Nesher, P. (1987). Towards an instructional theory: The role of students' misconceptions. *For the Learning of Mathematics*, 7(3):33–39.

Olivier, A. (1996). Handling pupils' misconceptions. *Pythagoras*, 21:10–19.

Radatz, H. 1979. Error analysis in mathematics education. *Journal for Research in Mathematics Education*, 10(3):163–172.

Riccomini, P.J. (2005). Identification and remediation of systematic error patterns in subtraction. *Learning Disability Quarterly*, 28(3):233–242.

Russell, M. & Masters, J. (2009). *Formative diagnostic assessment in algebra and geometry*. Paper presented at the annual meeting of the American Education Research Association. San Diego, California.

Sapire, I., Shalem, Y., & Reed, Y. (2013). *Assessment for Learning. Johannesburg: University of Witwatersrand and Saide.*

Shalem, Y., Sapire, I., & Sorto, M.A. (2014). Teachers' explanations of learners' errors in standardised mathematics assessments. *Pythagoras,* 35(1), Art. #254, 11 pages. http://dx.doi.org/10.4102/pythagoras.v35i1.254

Shulman, L.S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2):4–14.

Sousa, D.A. (2008). *How the brain learns mathematics*. California: Corwin Press.

Yang, C.W., Sherman, H. & Murdick, N. (2011). Error pattern analysis of elementary school-aged students with limited English proficiency. *Investigations in Mathematics Learning*, 4(1):50–67.

Topics

Number names and symbols

ANA 2013 Grade 3 Mathematics Items 11 and 12

11. Write the number name for 468.

_____

12. Write the number symbol for three hundred and sixty.

_____

What should a learner know to answer these questions correctly?

Learners should be able to:

- Write the number name for a given three digit number;
- Read a number name and represent the number in numerical form;
- Show an understanding of place value and correctly identify the value of given numbers.

Where is this topic located in the curriculum? Grade 3 Term 1 - 3

Content area: Numbers, Operations and Relationships.

Topic: Represent whole numbers.

Concepts and skills:

- Number symbols and number names.

Item 11

- Identify, recognise and read number symbols 0 - 500;
- Write number names 0 - 500.

Item 12

- Identify, recognise and read number names 0 - 500;
- Write number symbols 0 - 500.

What would show evidence of full understanding?

Item 11

- If the learner recognised the value of each digit correctly and wrote the answer 'four hundred and sixty eight' (Correct spelling is not necessary for evidence of full understanding).

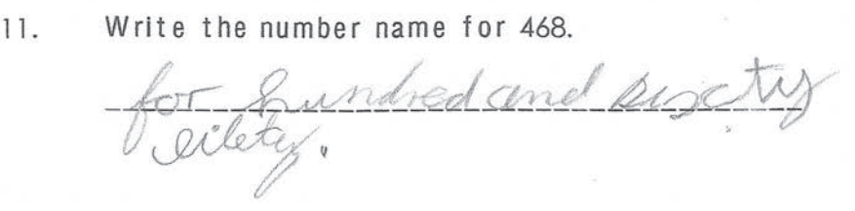
Item 12

- If the learner was able to read and understand the given number name and write the number symbol 360: this shows the learner understands the value of the digits and recognises that there are no ones in this number.

What would show evidence of partial understanding?

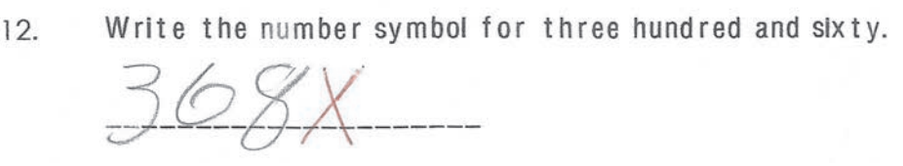
Item 11

- If the learner wrote the answer below:
  - The learner correctly identified the hundreds and the tens, but mistakenly referred to the 8 ones as eighty.
  - This may have been as a result of carelessness where the learner had just written 'sixty' and so continued in the same way, converting the ones into tens without intending to.



Item 12

- If the learner gave the answer 368:
  - This shows correct identification of the 300 and the 60.
  - The incorrect inclusion of the 8 may have been a result of carelessness.
  - The previous item referred to the number 468, so it is possible that the learner made an error in including the 8 ones as a carry-over from the previous question.



What would show evidence of no understanding?

Item 11

- If the learner wrote nothing or a number name that did not have any relation to the given number.

Item 12

- In the example below, the learner wrote '66' as the answer: this shows no understanding as the number symbol required for the answer was a three digit, not a two digit number.
- Whilst this learner did correctly read the word 'sixty', the learner did not recognise the three hundred.

- In addition, the learner incorrectly included six ones.
- This suggests that the learner has no real understanding of the value of the given numbers and is unable to correctly read a number name and write it as a number symbol.

12. Write the number symbol for three hundred and sixty.

What do the item statistics tell us?

Item 11

69% of learners answered the question correctly.

Item 12

48% of learners answered the question correctly.

Factors contributing to the difficulty of the items

Item 11

This item may have been difficult for learners as the writing of number names in many African languages involves numerous words which can be confusing.

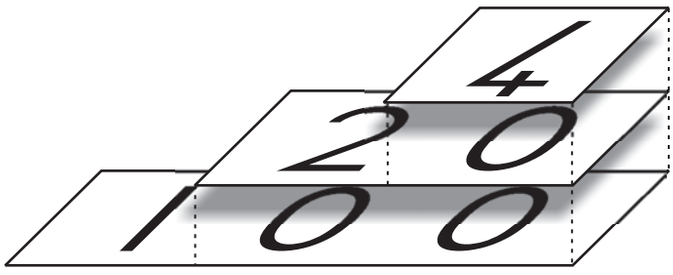
Item 12

This item may have been made more difficult for learners as the hundreds in Items 11 and 12 differed by only one and there were the same number of tens in both items. This may have resulted in an increased number of careless errors.

Teaching strategies

Reading number names

- Ask the learners to work in pairs or groups of four.
- Provide learners with place value cards **(see printables in item 1, p36)**.
- Explain to the learners that you will write a number name on the board and they must then find the corresponding numbers to build up the number symbol with their cut up place value cards.
- Write the number name 'one hundred and twenty-four' on the board.
  - Ask the learners to read the number out loud.
  - Encourage the learners to listen to the number as they read it and to identify the value of each number as they say the words.
  - Learners discuss the number symbol in their groups/pairs and then select the correct number symbol cards to make up the number.

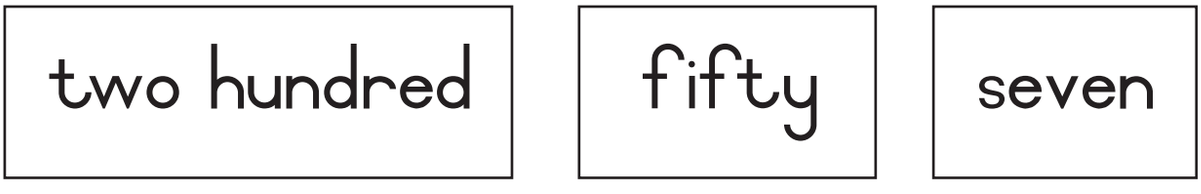


- Ask the learners to stack the place value cards to show the number '124' as it is written on the board.
- Ask learners to read the number aloud.
- Then ask learners to separate the place value cards so that they lie in a straight row, beginning with the 100 card.

- Say to the learners, “What can you tell me about the cards you have chosen?” The learners should respond with a variety of answers.
- Encourage learners to verbalise that each card represents the value of one of the digits in the number symbol '124'.

Reading number symbols

- Ask the learners to work in pairs or groups.
- Provide learners with number name cards **(see printables)**.
- Explain to the learners that you will write a number symbol on the board and they must then find the corresponding words to make up the number name with their cut up number name cards.
- Write the number symbol '257' on the board.
  - Ask the learners to read the number aloud.
  - Encourage the learners to listen to the number as they read it and to identify the value of each number as they say the words.
  - Learners discuss the number symbol in their groups/pairs and then select the correct number name cards to make up the number name.



- Ask the learners “Why did you choose these cards to make up the number name?” Learners should respond with a variety of answers.
- Encourage learners to realise that the '2' represents two hundred, the '5' represents fifty, and the

'7' is seven ones.

- An enlarged copy of the number name cards can be displayed on the wall for future reference.

Hundreds, tens and ones cards

- Ask the learners to work on white boards or scrap paper.
- Ask three learners to come to the front of the classroom and give each a 'hundreds', 'tens' or 'ones' flashcard to hold **(see printables)**.
  - Ask the learner holding the 'hundreds' card to call out a number. The learner could call out '3', for example.
  - Ask the learner holding the 'tens' card to call out a number. The learner could call out '9', for example.
  - Ask the learner holding the 'ones' card to call out a number. The learner could call out '7', for example.
  - The learners seated at their desks must then use these numbers to write the number name and the number symbol, for example 'three hundred and ninety-seven' and '397'.
  - Ask the three learners with the flashcards to go back to their desks and ask another three learners to come to hold the flashcards.
  - Repeat the above steps, asking the learners with the flashcards to call out different numbers so that the class has multiple opportunities to write out different numbers.

Other examples of how number names and symbols can be tested

ANA 2014 Grade 3 Mathematics Item 11

11. Write down the number symbol for three hundred and thirty-six.

_____

ANA 2014 Grade 3 Mathematics Item 12

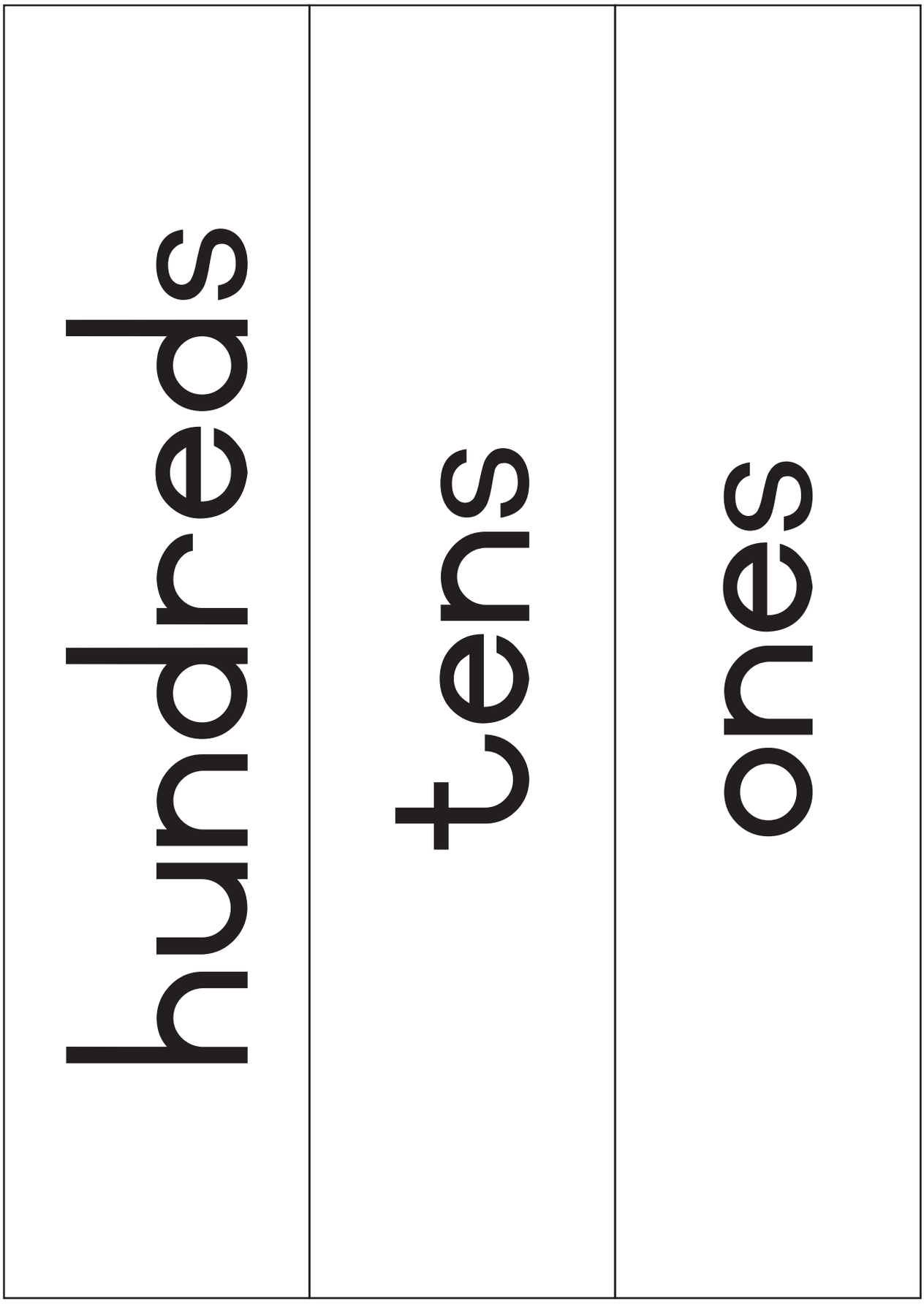
12. Write down the number name for 165.

_____

Printable: Number name cards

one hundred	ten	eleven	one
two hundred	twenty	twelve	two
three hundred	thirty	thirteen	three
four hundred	forty	fourteen	four
five hundred	fifty	fifteen	five
six hundred	sixty	sixteen	six
seven hundred	seventy	seventeen	seven
eight hundred	eighty	eighteen	eight
nine hundred	ninety	nineteen	nine

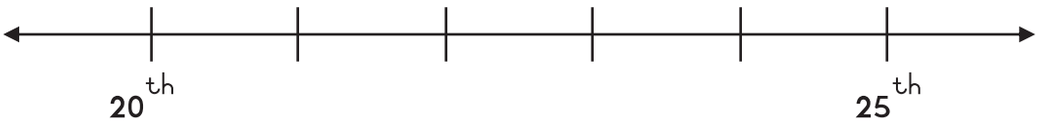
Printable: Place value flash cards



Number concept: ordinal numbers

ANA 2013 Grade 3 Mathematics Item 27

27. Fill in the missing ordinal numbers on the number line



What should a learner know to answer this question correctly?

Learners should be able to:

- Read the question with understanding;
- Understand the abbreviation of 20th (twentieth) and 25th (twenty fifth);
- Have knowledge and understanding of the abbreviations to use for all numbers between 20th and 25th;
- Understand basic mathematical vocabulary such as: cardinal numbers, ordinal numbers;
- Understand and determine the position of a number.

Where is this topic located in the curriculum? Grade 3 Term 3

Content area: Numbers, Operations and Relationships.

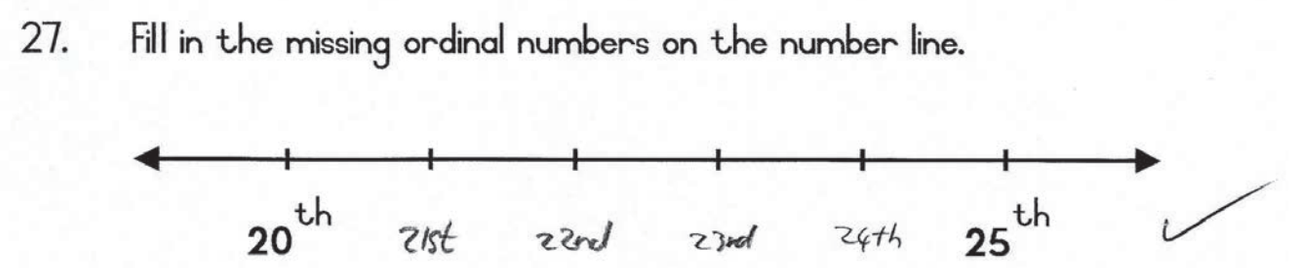
Topic: Describe, compare and order numbers.

Concepts and skills:

- Use ordinal numbers to show order, place and position;
- Use, read and write ordinal numbers, including the abbreviated form, from 1st up to 31st.

What would show evidence of full understanding?

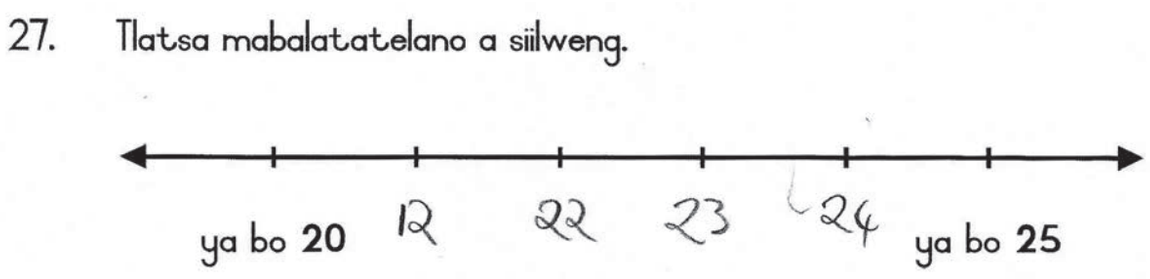
If the learner knew and wrote the ordinal numbers 21st, 22nd, 23rd and 24th with the correct abbreviation in his/her home language: this shows the learner knows and understands the correct order and position of numbers.



What would show evidence of partial understanding?

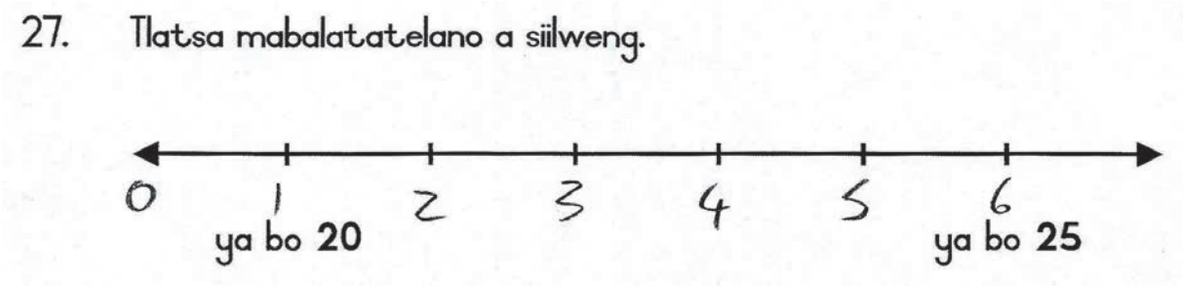
If the learner wrote the correct cardinal numbers but not the ordinal numbers:

- In the example that follows the first number in the answer is incorrect; the following numbers do form the correct sequence of counting numbers from 21, but the learner did not give the ordinal numbers as required by the question.
- If the learner wrote down ordinal numbers, but the incorrect ordinal numbers, he or she realised that the question required ordinal numbers, but did not give the right ones.



What would show evidence of no understanding?

If the learner wrote down the incorrect cardinal numbers on the number line.



What do the item statistics tell us?

49% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Learners may not be completely familiar with the abbreviations of ordinal numbers as required;
- Leaners may not understand what ordinal numbers are;
- Learners may not understand the difference between cardinal and ordinal numbers.

Teaching strategies

Comparing and ordering ordinal numbers from 1st to 10th

- The teaching of ordinal numbers involves a lot of mathematical vocabulary that learners need to understand in order to fully understand the concept.

- Once learners have learned cardinal numbers, you will have to introduce ordinal numbers.
- The teaching of ordinal numbers should not directly follow after the teaching of cardinal numbers because learners need a lot of practice with cardinal numbers before being introduced to ordinal numbers.
- If learners have had enough practice with cardinal numbers, the teaching of ordinal numbers should be fun and easy.
- Since **ordinal numbers have nothing to do with amount but rather relate to position**, do not start off by using numbers; instead use images of other types of items and words.

Activity 1

Simulate a race on the board.

- Draw a chart like this on the board:

									
first	second	third	fourth	fifth	sixth	seventh	eighth	ninth	tenth

  - Referring to the chart, discuss with your learners the positions of each item by saying, for example "*The house is first and the hat is second.*"
  - Begin with just a few words or images on the board and work your way up to give learners practice using the first ten ordinal numbers.
  - Introduce the words *'first'* and *'last'*,since these are position related vocabulary words.
  - For example: The *'ball is last'* and the *'house is first'*
  - Positional vocabulary that learners should also be familiar with includes: before, after, between, next to, on the right, on the left.
- Once learners understand the meaning of these words, you can use the number and ordinal name flashcards to practice pronunciation of ordinal numbers.

1st	3rd	5th	7th	9th
2nd	4th	6th	8th	10th
first	third	fifth	seventh	ninth
second	fourth	sixth	eighth	tenth

- Ensure that the ordinal numbers from first to tenth are revised before proceeding to higher ordinal numbers.
- A lot of practical exercise is needed for learners to master ordinal numbers, as understanding ordinal numbers is based on language development.

Activity 2

- Ask ten learners to stand next to each other.
  - Ask the third child to step forward. Discuss how learners would work out who is the third child.
  - Continue until learners understand the positions of first to tenth.
  - Also ask which learner is first and which learner is last.

- Following this activity learners can be prompted to write sentences about themselves such as

I am the first child in my family.

- You can also ask learners to discuss what they do on a daily basis by asking questions such as:
  - "What's the first thing you do in the morning?"
  - "What's the first thing you do after school?"

Activity 3

- Make a set of ordinal number and picture cards **(see printables)**.

									
1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
first	second	third	fourth	fifth	sixth	seventh	eighth	ninth	tenth

- Learners can use the ordinal number cards to play games to practice the words relating to ordinal numbers.

Recommended games:

Snap

- Ask learners to work in pairs.
- Make sure that the ordinal number names and ordinal abbreviations are cut up and shuffled, and that you have a set of cards for each pair.
- One learner in each pair deals out the pack of shuffled cards so that each learner has an equal number of cards.
- The learners hold their cards face down so that they cannot see what is written on them.
- One learner then places his/her top card face up on the desk.
- The other learner follows by placing his/her top card face up on top of the first card.
- As soon as two consecutive cards are a match (for example: 'third' and '3rd') the learners must say "snap" as quickly as possible.
- The learner who says "snap" first collects all the face up cards off the desk and adds them to his/her pack.

- The learner with all cards at the end wins the game.

Memory game

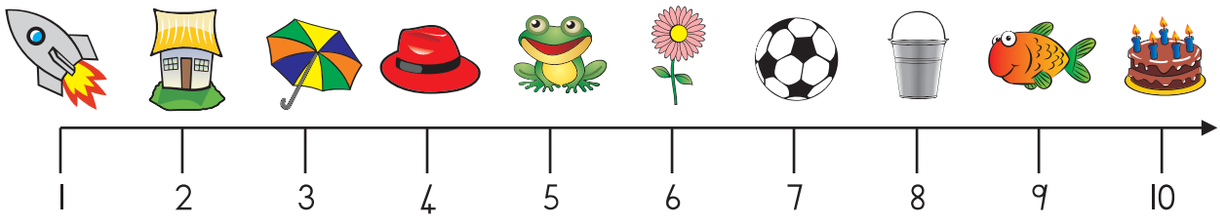
- Ask learners to work in groups of four.
- Make two copies of the **printable** for each group playing the game.
- Use one copy as a key for the learners to refer to during the game (do not cut it up).
- Cut up the other copy so that the pictures, ordinal number names and ordinal abbreviations can be shuffled.
- One learner lays out all the cards face down on the desk in a random arrangement.
- Each learner in the group is then given an opportunity to turn over 3 cards, in an attempt to create a matching set of three cards. For example: The hat, 5^{th} and fifth.
- Learners can refer to the key to ensure they have the matching cards.
- When a matching set of three is found, the learner may remove the cards from those laid out on the desk and have another turn.
- If the three cards that are turned over do not match, then all three of the cards must be placed face down on the desk again, in the same place as before (even if two of the three cards do match).
- Learners keep taking turns to turn over the cards until all the matching sets have been found.
- The learner with the most matching sets at the end wins the game.

Comparing and ordering ordinal numbers from 1^{st} to 31^{st}

- Once learners understand ordinal numbers from first to tenth, explain how we write **ordinal numbers**.
- Make sure that learners know that, except for '*eleventh'*, '*twelfth'*, and '*thirteenth'*, numbers ending with 'one', 'two', or 'three' are irregular and should be said '*first'*, '*second'*, and '*third'* respectively.
- As with cardinal numbers, learners may confuse words such as 'thirteenth' and 'thirtieth', so these may require a lot of practice.
- Grade 3 learners should know ordinal numbers from first up to thirty-first **(see printables)**.

Teaching ordinal numbers using a number line

- Once learners understand that **ordinal numbers have nothing to do with amount, but rather show the position of a number** you can introduce number lines to consolidate the learners' understanding of ordinal numbers.
- Learners should practice writing ordinal number names and abbreviations correctly.

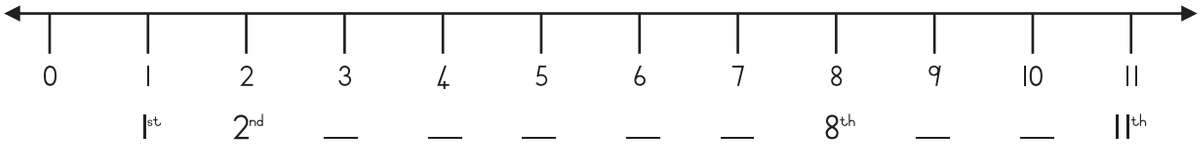


- Discuss the position of the items on the number line by asking questions such as:
  - The rocket is? Answer: First or 1st.
  - The hat is? Answer: Fourth or 4th.
  - The cake is? Answer: Tenth or 10th.

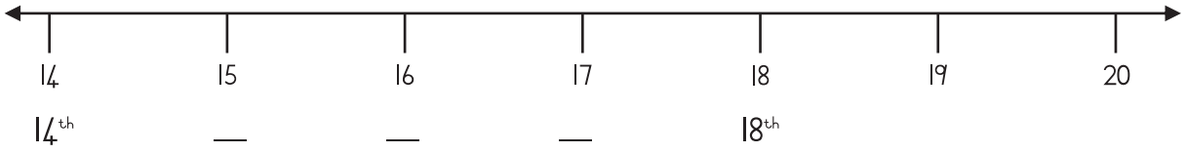
Activity 4

- Explain to learners how to use the number line to fill in the missing ordinal numbers.

1. Fill in the missing ordinal numbers.



2. Fill in the missing ordinal numbers:



3. Fill in the missing ordinal numbers:

- Use the written number line activity to consolidate **(see printables).**

Teaching ordinal numbers using a calendar

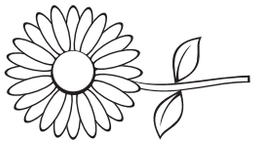
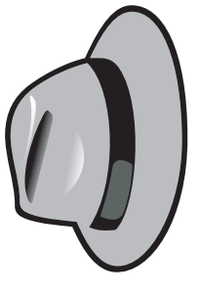
- The weather chart is a very valuable resource in the Foundation Phase class for teaching and consolidating ordinal numbers on a daily basis (Use a current weather chart in your class. The one below is given as an example).
- Lessons like this can consolidate calendar vocabulary as well as ordinal number concept.

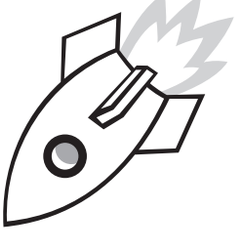
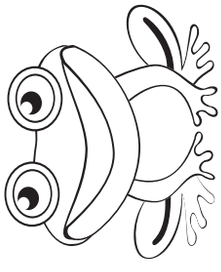
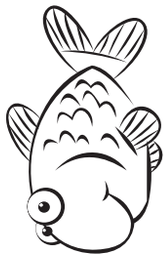
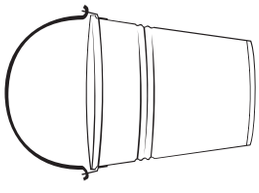
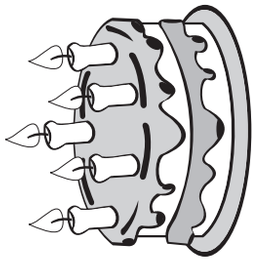
- Weather chart: January 2014

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

- Repeat the days of the week: Sunday, Monday, Tuesday, Wednesday, Thursday, Friday and Saturday.
- Repeat the weather chart vocabulary: sunny, cloudy, partly cloudy, and rainy.
- Use the calendar to indicate the days of the week, the date and discuss the weather of the day;
- Let a learner place the correct weather symbol on the calendar.
- Examples of questions to be asked based on the calendar:
  - Which day is it today? Monday.
  - What is the date today? It is the 21st.
  - Which day was it yesterday? Sunday.
  - What was the date yesterday? It was the 20th.
- The weather chart should be used daily to consolidate ordinal numbers in an informal manner.

Printable: Ordinal number name, abbreviation and picture cards

	1st	first
	2nd	second
	3rd	third
	4th	fourth
	5th	fifth

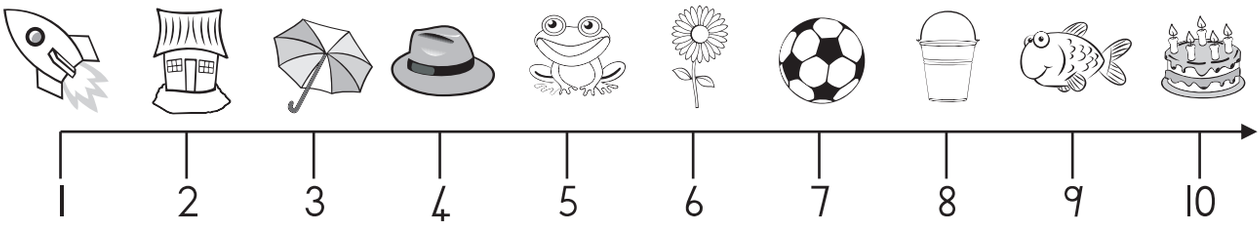
	6th	sixth
	7th	seventh
	8th	eighth
	9th	ninth
	10th	tenth

Printable: Ordinal numbers to thirty-first

1	1st	first
2	2nd	second
3	3rd	third
4	4th	forth
5	5th	fifth
6	6th	sixth
7	7th	seventh
8	8th	eighth
9	9th	ninth
10	10th	tenth
11	11th	eleventh
12	12th	twelfth
13	13th	thirteenth
14	14th	fourteenth
15	15th	fifteenth
16	16th	sixteenth
17	17th	seventeenth
18	18th	eighteenth
19	19th	nineteenth
20	20th	twentieth
21	21st	twenty-first
22	22nd	twenty-second
23	23rd	twenty-third
24	24th	twenty-fourth
25	25th	twenty-fifth
26	26th	twenty-sixth
27	27th	twenty-seventh
28	28th	twenty-eighth
29	29th	twenty-ninth
30	30th	thirtieth
31	31st	thirty-first

Printable: Ordinal numbers

Position of numbers: Fill in the ordinal numbers:



Answer the questions:

The frog is _____

The red hat is _____

The cake is _____

The ball is _____

The house is _____

What picture is fourth in the row? _____

What picture is between the seventh and the ninth pictures? _____

Place value

ANA 2013 Grade 3 Mathematics Items 2 and 13

2. Break down the number 489.

   A 80 + 900 + 4

   B 400 + 80 + 9

   C 90 + 40 +80

   D 800 + 90 + 40

13. Write the value of the underlined digit in the number 7̲54.

    _____

What should a learner know to answer these questions correctly?

Learners should be able to:

Item 2

- Understand the concept of place value and be able to determine the hundreds, tens and ones of a given number;
- Break down a number into its composite parts and identify what each digit represents.

Item 13

- Understand the concept of place value and be able to correctly identify and state the hundreds, tens and ones of a given number.

Where is this topic located in the curriculum? Grade 3 Term 3

Content area: Numbers, Operations and Relationships.

Topic: Place value.

Concepts and skills:

- Recognise the place value of numbers to 750.

Item 2

- Know what each digit represents;
- Decompose three-digit numbers up to 750 into multiples of hundreds, tens and ones/units.

Item 13

- Identify and state the value of each digit.

What would show evidence of full understanding?

Item 2

- If the learner gave the answer B (400 + 80 + 9);
- This shows the learner's ability to break down a number into hundreds, tens and ones/units.

Item 13

- If the learner correctly identified the 7 in the number 754 as representing 700.

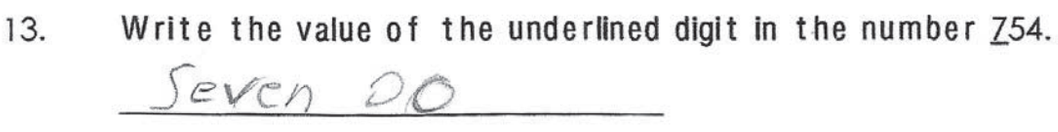
What would show evidence of partial understanding?

Item 2

- If the learner selected answer A (80 + 900 + 4);
- The learner correctly identified the 8 in the number 489 as representing 80;
- However, the learner was unable to correctly identify the hundreds and the units in the number.

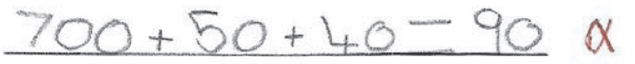
Item 13

- If the learner correctly identified the 7 as being in the hundreds, but struggled to state this clearly;
- This shows that whilst an understanding of the value of the digit 7 is evident, the correct method of stating this number ('seven hundred' / '700' / 7H / 7 hundred) is under-developed.



- In the example below the learner decomposed the number 754 into 700 + 50 + 40 = 90.
- This shows partial understanding as the 7 was correctly identified as representing 700, although the 4 was incorrectly identified as a ten.
- The learner added the three numbers (700 + 50 + 40) together to get a total of 90, which indicates that the learner has a limited understanding of place value. However, this was not the focus of the question.

13. Write the value of the underlined digit in the number 754.



What would show evidence of no understanding?

Item 2

- If the learner selected answer C (90 + 40 + 80) or D (800 + 90 + 40), this shows evidence of little to no understanding of place value:
- In answer C (90 + 40 + 80), although the number 8 is correctly identified as representing 80, the numbers 4 and 9 are also seen as representing tens. This suggests that the learner does not understand the concept of place value and cannot differentiate between the three numbers in the number 489.
- In answer D (800 + 90 + 40), the learner was not able to identify or differentiate between the numbers 4 and 9, seeing them both as tens. The learner showed no recognition of units/ones in this number, and the 8 was incorrectly viewed as representing 800, rather than 80.

Item 13

- If the learner gave no response this may be due the learner's lack of understanding of place value;
- If the learner identified the 7 as representing 70 or 7 this also shows a lack of understanding of place value.

What do the item statistics tell us?

Item 2

74% of learners answered the question correctly.

Item 13

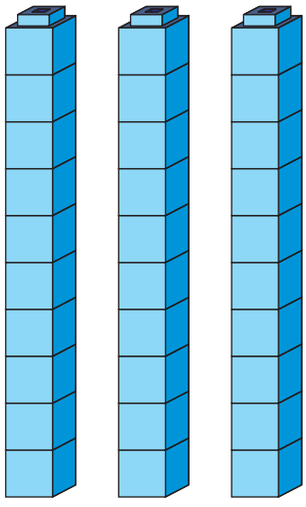
50% of learners answered the question correctly.

Factors contributing to the difficulty of the items

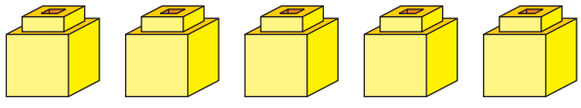
- Item 2 may have been made more difficult by the fact that the digits in the number 489 go from smallest to biggest as you read from left to right, whereas their values go from biggest to smallest in the same direction.
- Item 13 may have been made more difficult by the fact that the underlined number is in the hundreds. Some learners may not be comfortable working with three digit numbers yet and so struggled to determine the value of this number.

Teaching strategies

Establishing place value concept – using Unifix blocks



- Work with children at a concrete level to establish their understanding of place value.
- Give the learners a 2 digit number, such as 35.
- Write the number on the board and ask the learners to read the name "thirty five" out loud. This consolidates number recognition.
- Ask the learners: "What does the 3 tell us?" and "What does the 5 tell us?"
- Encourage learners to discuss the value of these digits and model the correct language.
- Remind children that the 3 represents "3 bundles of ten", and show them 3 towers of 10 Unifix blocks.
- Ask the learners "Do we have enough ones to make up another bundle of 10?"
- Then show them that there are 5 loose Unfix blocks and say "We call these 5 ones or units".

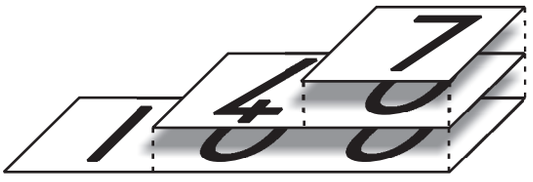


- Repeat this with multiple examples, encouraging learners to establish how many bundles of ten and how many ones there are in each number.
- Once the learners are comfortable with two digit numbers, then introduce 3 digit numbers.

Establishing place value concept – using place value cards

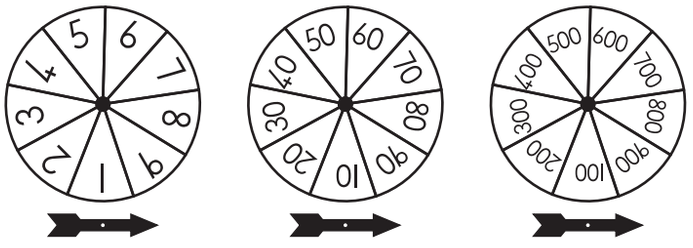
- Learners can work in pairs or groups of 4, using the place value/Flard cards **(see printables, Item 1, p.36)**.
- Ask learners to arrange the ones cards in order, from smallest to largest: 1, 2, 3, 4, 5, 6, 7, 8, 9
- Repeat with the tens and hundreds cards as well: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 200, 300, 400, 500, 600, 700, 800, 900
- Write the number 147 on the board and ask learners to discuss the value of each digit in their groups.
- Learners can discuss and select the correct cards that they feel represent the value of each digit.
- Learners may select incorrect cards, such as 1, 4 and 7 instead of 100, 40 and 7

- Discuss the value of the digits and show learners how to stack the place value cards so that the place holders are covered correctly.



Consolidating place value concept – using spinners

- Learners can work in pairs or groups of 4, using the place value spinners **(see printables)**.
- The spinners are used to generate numbers with three places – units, tens and hundreds.
- Learners spin each of the spinners, noting the numbers that the spinners land on.



- Learners record these numbers in columns in the place value table **(see printables)** to differentiate between the place values of the numbers.

  For example:

hundreds	tens	ones
3	6	5

- Learners then use these digits to write a number sentence and give the total value.

  For example: The learner who got the numbers in the table above after spinning would write:

  $300 + 60 + 5 = 365$

Other examples of how place value can be tested

ANA 2014 Grade 3 Mathematics Item 6

6. Break down the number 254 into hundreds, tens and units.

A 200 + 50 + 4

B 200 + 5 + 4

C 200 + 5 + 40

D 200 + 50 + 40

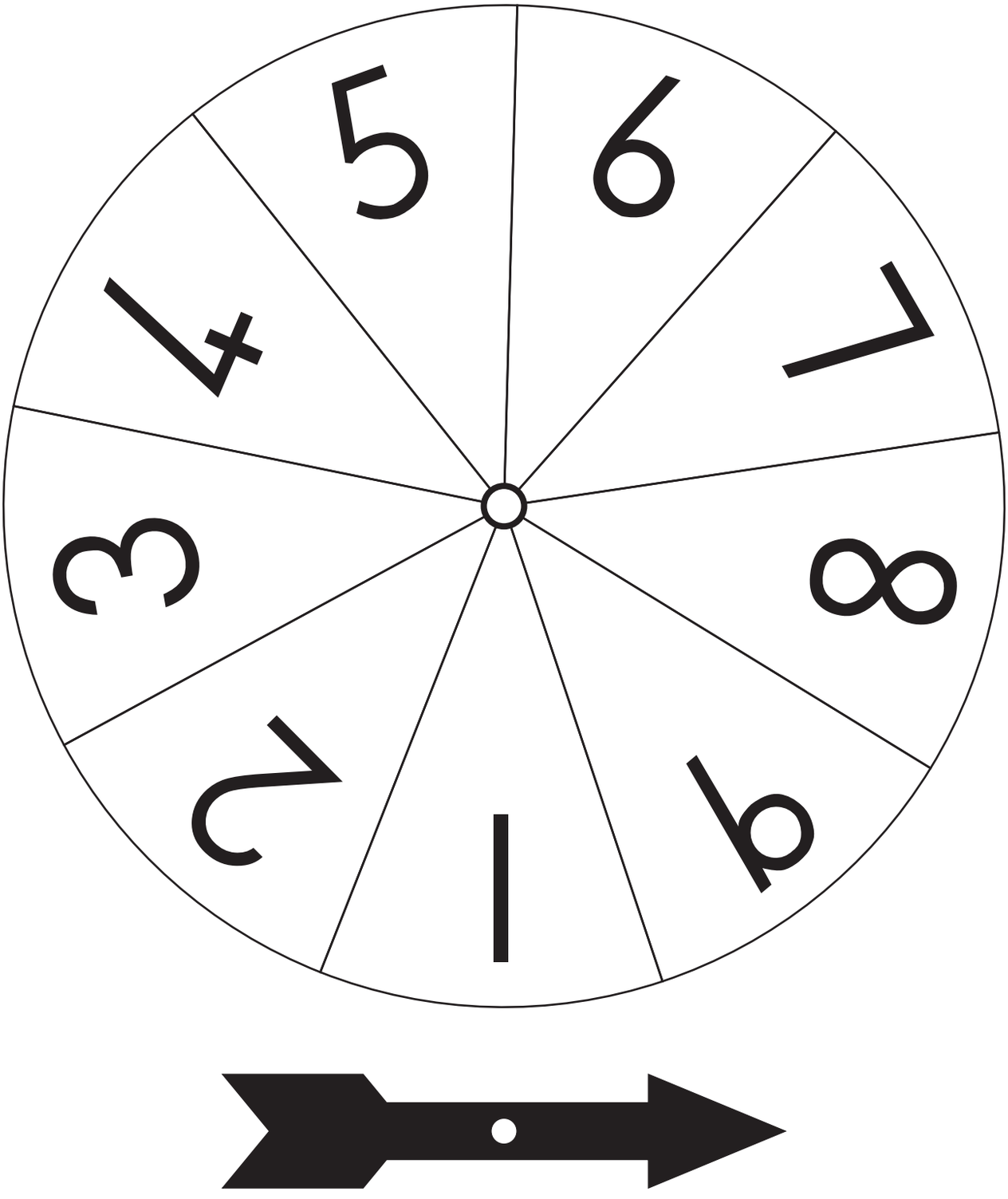
ANA 2014 Grade 3 Mathematics Item 13

13. Write down the value of the underlined digit in the number:

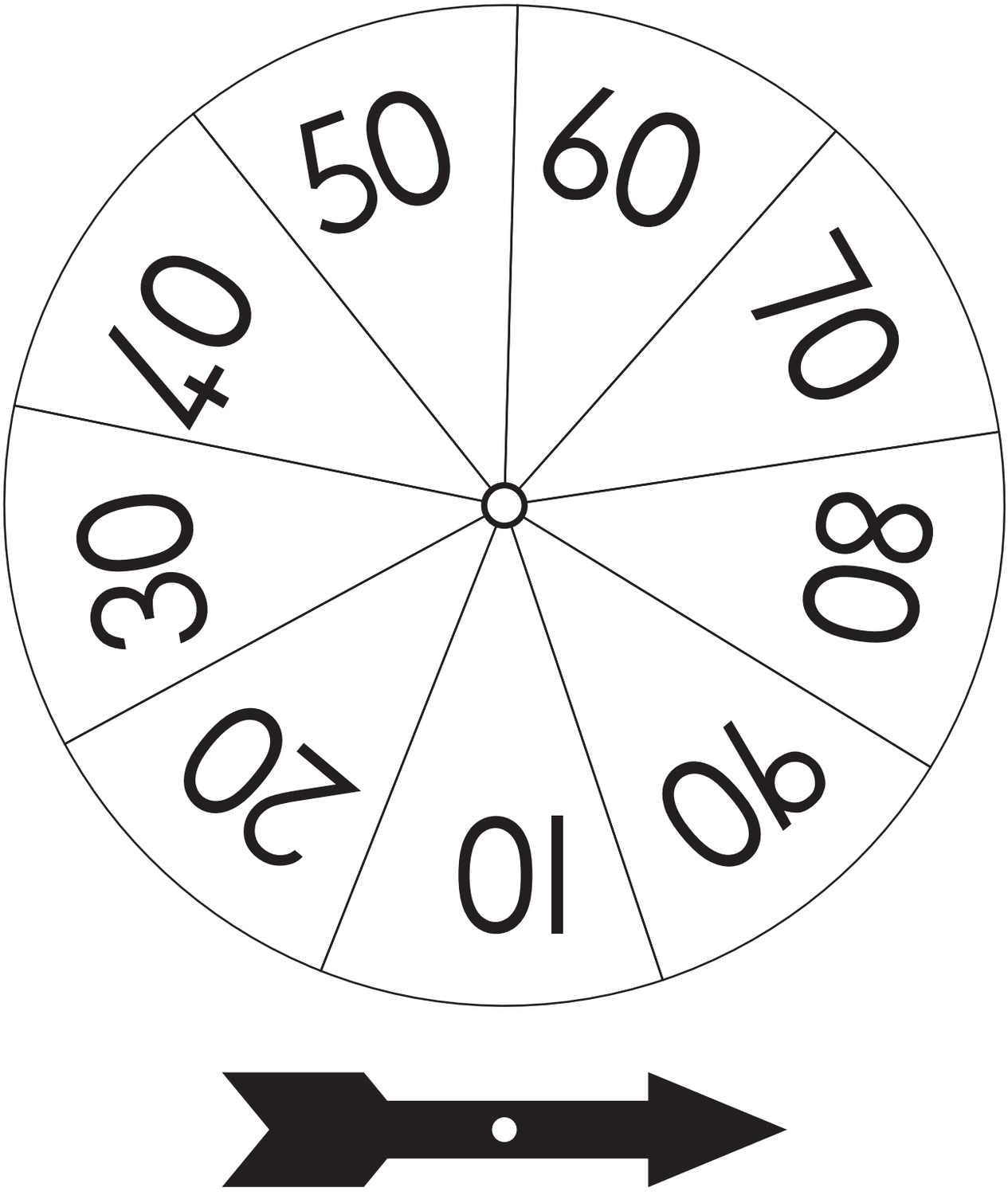
4̲72 _____

Notes:

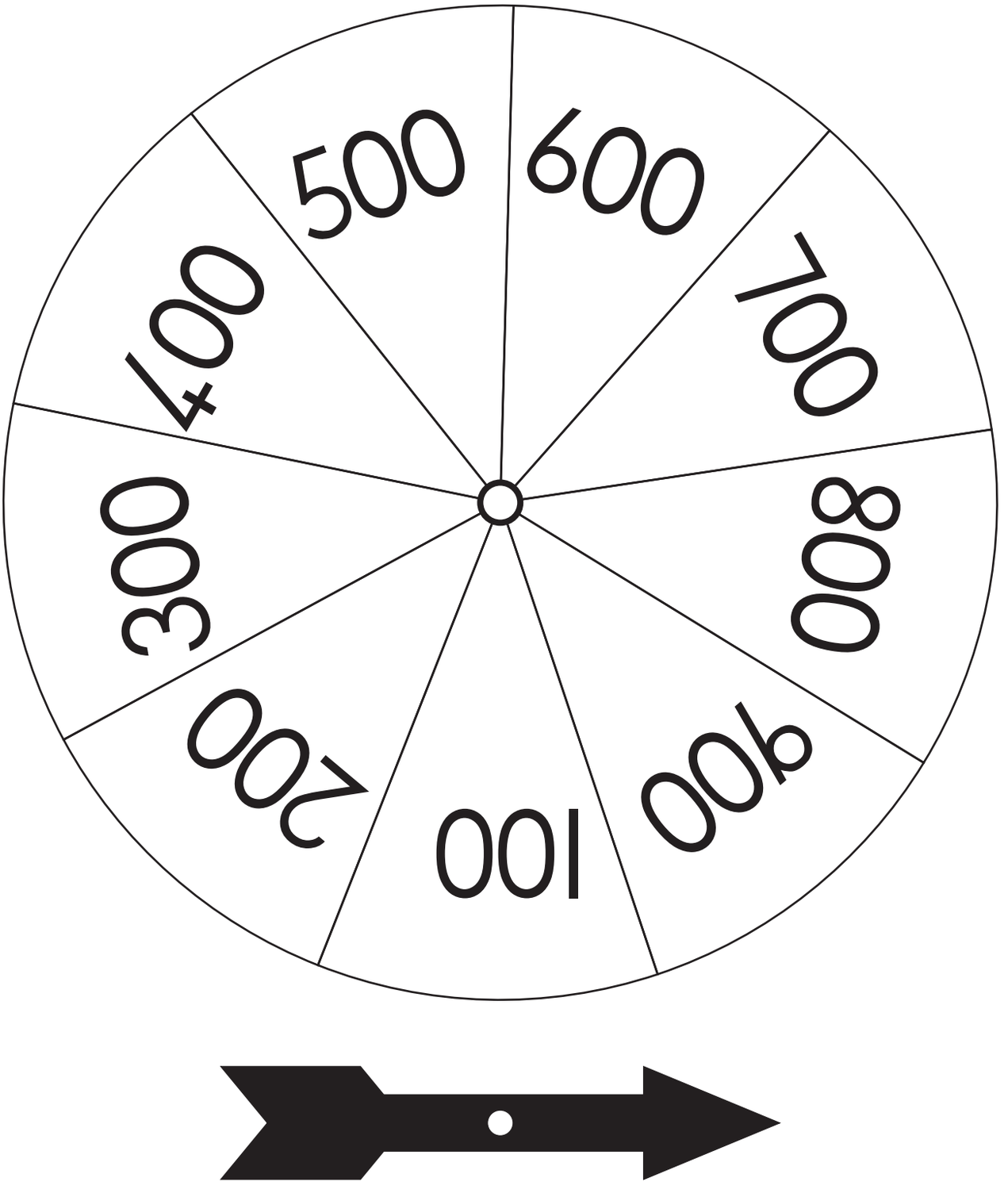
Printable: Place value spinner (units)



Printable: Place value spinner (tens)



Printable: Place value spinner (hundreds)



Printable: Place value table

hundreds	tens	ones

Number concept: 3-digit numbers

ANA 2013 Grade 3 Mathematics Item 1

1. Arrange 172, 217, 127, 712 from the smallest to the greatest.

   A 217, 127, 721, 172

   B 721, 217, 172, 127

   C 172, 127, 721, 217

   D 127, 172, 217, 712

What should a learner know to answer this question correctly?

Learners should be able to:

- Recognise the smallest and greatest numbers;
- Understand the mathematical vocabulary, e.g. arrange, smallest, greatest;
- Understand place value of 3 digit numbers, i.e. hundreds, tens, units;
- Understand how to identify the smallest or greatest number by looking at the first digit (hundreds) and then tens and units;
- Apply skills to identify distractors in multiple choice questions (elimination of incorrect answers).

Where is this topic located in the curriculum? Grade 3 Term 1

Content area: Number, Operations and Relationships.

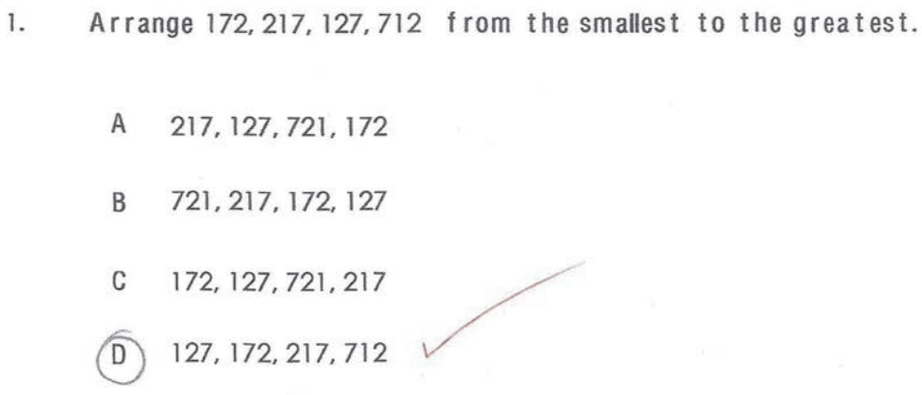
Topic: Describe, compare and order numbers.

Concepts and skills:

- Describe and order whole numbers up to 999 from smallest to greatest and greatest to smallest.

What would show evidence of full understanding?

- To show evidence of full understanding the learner ought to indicate the correct answer, D.



What would show evidence of partial understanding?

- If the learner gave the answer B, this indicates that the learner can order numbers but selected the correct answer for greatest to smallest and not smallest to greatest.

What would show evidence of no understanding?

- If the learner selected option A or C (this indicates the learner's incorrect use of place value in ordering the numbers);
- If the learner indicated more than one response to the question;
- If the learner made no attempt to select the correct answer.

What do the item statistics tell us?

51% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Learners not understanding the correct mathematical vocabulary, smallest and greatest;
- Learners having not fully mastered the value of digits in numbers.

Teaching strategies

Describe and order whole numbers up to 999

- Ensure learners fully understand the numerosity of one-digit numbers.
- Proceed to two-digit numbers.
- In grade 3 learners should be exposed to the numerosity of three-digit numbers in order for them to understand the concept of smallest/greatest and greatest/smallest numbers.

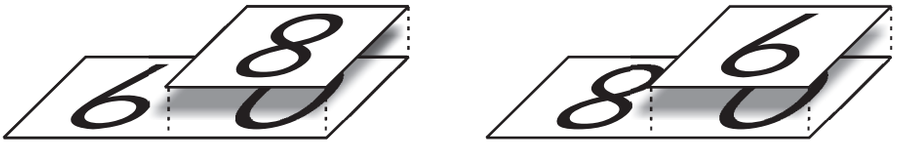
Consolidate number symbols 0 - 99

- Call out a number between 0 and 99. Learners point to the number symbols on their number charts as the teacher calls out the numbers.
- Learners should be given the opportunity to read out numbers. Oral reading of numbers helps consolidate number recognition and naming.
- Call out two numbers, 68 and 86, and ask the learners to tell you which number is the biggest/greatest and which is the smallest. Learners should explain why the number chosen is the greatest or smallest. The explanation should bring out the place value of the digits in the numbers since the numbers' place value determines their relative size **(see printables)**.

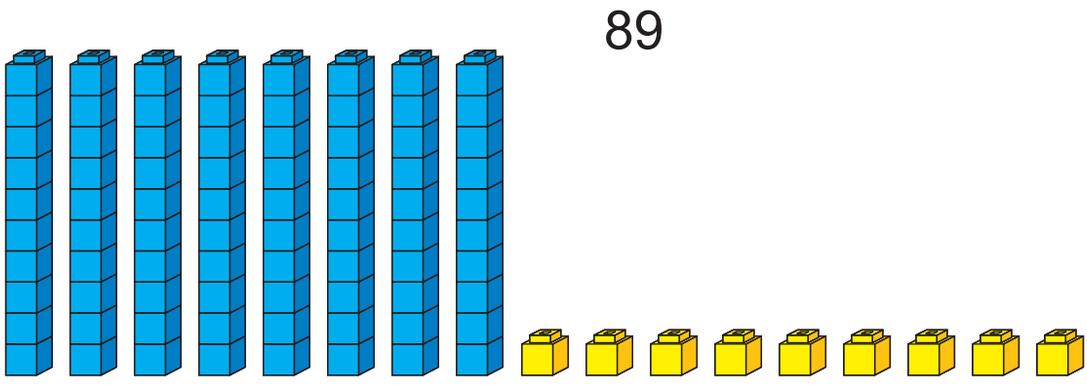
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

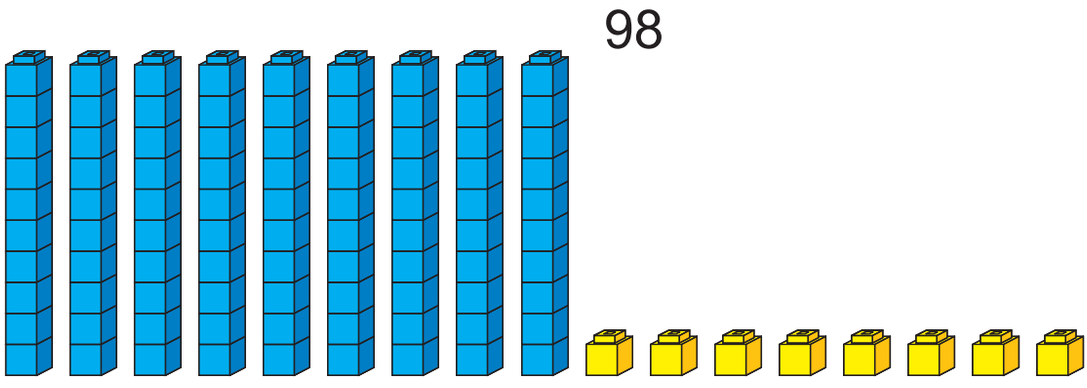
- Ask learners to indicate which of the two tens is bigger/greater and which is smaller. Explain to learners that the digit in the tens place indicates the size of a two-digit number. Once the “bigger ten” is identified, the relative sizes of the numbers will be known. If the tens are the same, then the units should be used to determine the bigger or smaller number.
- Let learners build 68 and 86 with their place value/Flard cards. Show learners that numbers 68 and 86 are built using the two-digit numbers 60 or 80 and the one-digit number 6 or 8:

$68 = 60 + 8$ and $86 = 80 + 6$



- Let learners use their place value/Flard cards to build the numbers you asked for, e.g. 79, 57, 87, 97. Ensure learners understand the place value of the digits in each number. Learners must be able to identify which number is bigger or smaller than a given number. Ask the learners to tell you which number is bigger and which number is smaller than …, e.g. which number is bigger than 87, which number is smaller than 87?
- Let learners show the difference between 89 and 98 using their Unifix cubes to display the numbers. The learners must understand that in the case of two-digit numbers the tens determine if one number is bigger than another. If learners physically put out the two numbers using their Unifix cubes, they can see that the 89 is smaller in quantity than 98 because it has fewer tens.





- Give learners four two-digit numbers and ask them to order the numbers from the smallest number to the greatest number and vice versa. Ensure the numbers have distracters in order to make sure the learners understand the value of the numbers, e.g.

78	87	97	79

- Learners should be encouraged to write the order of the numbers above the given series of numbers before they write out their answers, for example:

(1)	(3)	(4)	(2)
78	87	97	79

- Final order, from smallest to greatest:

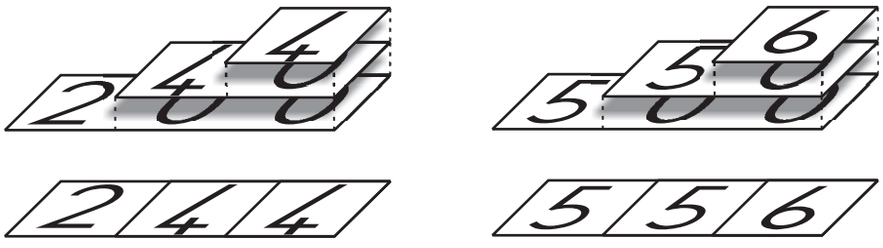
  78, 79, 87, 97

- Once learners understand the order of two-digit numbers you can move on to the teaching of three-digit numbers.

Consolidate number symbols 0 - 999

- Call out the numbers 468 and 886 and ask learners which number is bigger/greater and which is smaller? Learners should explain why the number they have identified is greater or smaller.
- You can allow learners to use place value/Flard cards to build up numbers and order them. Let learners build 244 and 556 with their place value/Flard cards **(see printables)**.
- Show learners that the numbers 244 and 556 are built using 200, 40 and 4 and 500, 50 and 6.

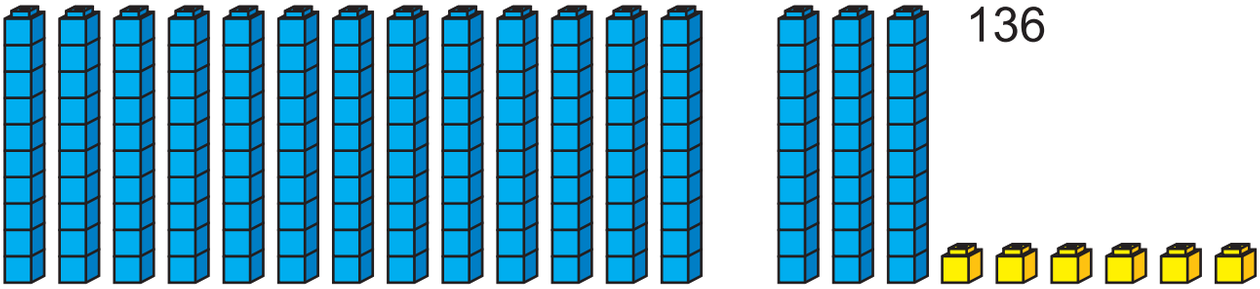
244 = 200 + 40 + 4 and 556 = 500 + 50 + 6

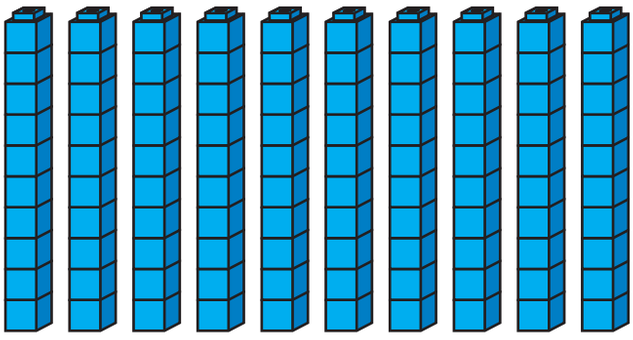


- For a 3-digit number, first focus on the digit in the hundreds place. Explain to the learners that the hundred indicates the size of a three-digit number. Ask the learners which hundred is bigger/greater and which hundred is smaller. If the hundreds are the same, then the tens are used to determine the bigger or smaller number and the same question is asked for the tens. If the hundreds and the tens are the same, the units are then used to determine which is the bigger or smaller number and the question is asked for the units.

- Let learners use their place value/Flard cards to build numbers you ask for, e.g. 739, 587, 857, 937. Ensure learners understand the place value of the digits in each number. Learners must be able to identify which number is bigger or smaller than a given number. Ask which number is bigger and which number is smaller than …, e.g. which number is bigger than 857, which number is smaller than 857?

- Let learners show the difference between 136 and 336 using their Unifix cubes to display the numbers. The learners must understand that in the case of three-digit numbers the hundreds determine if one number is bigger than another. If learners physically put out the two numbers using their Unifix cubes they can see that 136 is smaller in quantity than 336 because it has fewer hundreds.





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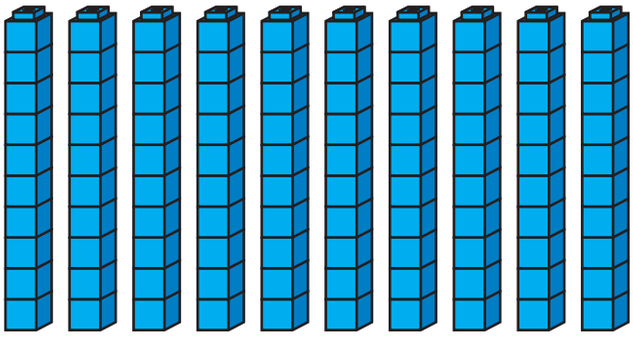
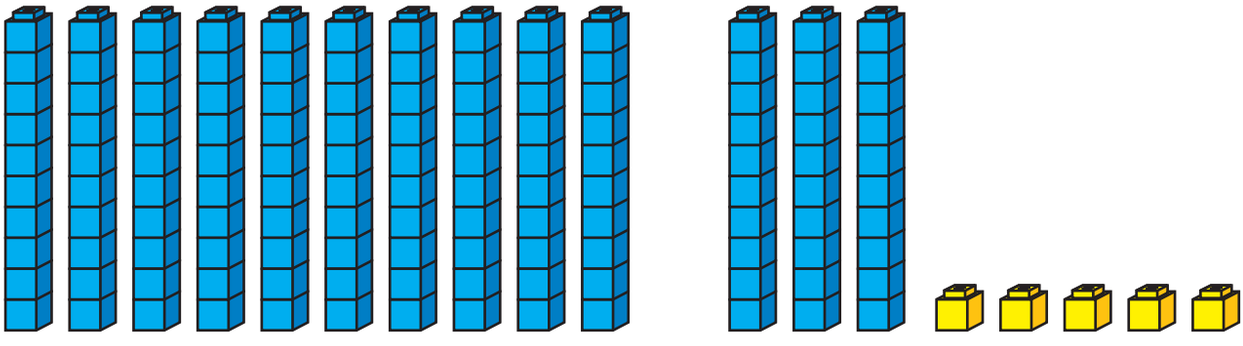




- Give learners four three-digit numbers and ask them to order the numbers from the smallest number to the greatest number and vice versa. Ensure the numbers have distracters in order to make sure the learners understand the value of the numbers, e.g.

378	837	987	879

- Remind learners to write the order of the numbers above the given series of numbers before they record their final answers.

Another example of how number concept can be tested

ANA 2014 Grade 3 Mathematics Item 1

1. Arrange 432, 324, 243, 342 from the smallest to the greatest.

   _____, _____, _____, _____

Notes:

Printable: Hundred charts

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Printable: Place value/Flard cards

900	90	9
800	80	8
700	70	7
600	60	6
500	50	5
400	40	4
300	30	3
200	20	2
100	10	1

Printable: Comparing and ordering numbers

Look at the two numbers. Is the number at the top greater or smaller than the number at the bottom? Colour the correct block next to the number

1.

123	greater	smaller
132		

2.

379	greater	smaller
739		

3.

456	greater	smaller
546		

4.

412	greater	smaller
214		

5.

560	greater	smaller
506		

6.

853	greater	smaller
835		

Order the numbers from the **smallest** to the **greatest** number

1.

378	831	381	879

2.

623	236	632	326

Order the numbers from the **greatest** to the **smallest** number

1.

378	831	381	879

2.

623	236	632	326

Calculation strategy: breaking down

ANA 2013 Grade 3 Mathematics Items 9.1 and 9.2

9. Calculate each of the following by using the 'breaking-down' method:

   9.1 795 - 213

   9.2 584 + 137

What should a learner know to answer this question correctly?

Learners should be able to:

- Recognise the value of each digit in a three digit number;
- Write out a number using expanded notation;
- Solve addition and subtraction problems using the breaking down method;
- Judge the reasonableness of their answers.

Where is this topic located in the curriculum? Grade 3 Term 1 - 3

Content area: Numbers, Operations and Relationships. (Context-free calculations).

Topic: Techniques (methods or strategies).

Concepts and skills:

- Use the following techniques when performing calculations: building up and breaking down of numbers.

Grade 3 Term 3

Content area: Numbers, Operations and Relationships (Context-free calculations).

Topic: 1.13 Addition and subtraction

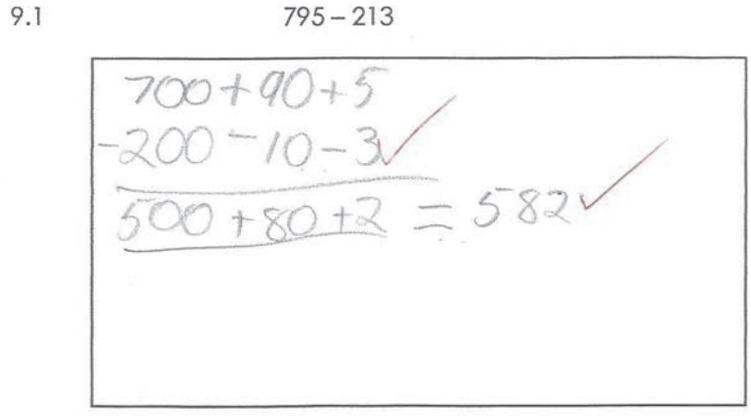
Concepts and skills:

- Add up to 800;
- Subtract from 800;
- Use appropriate symbols (+, –, =, □).

What would show evidence of full understanding?

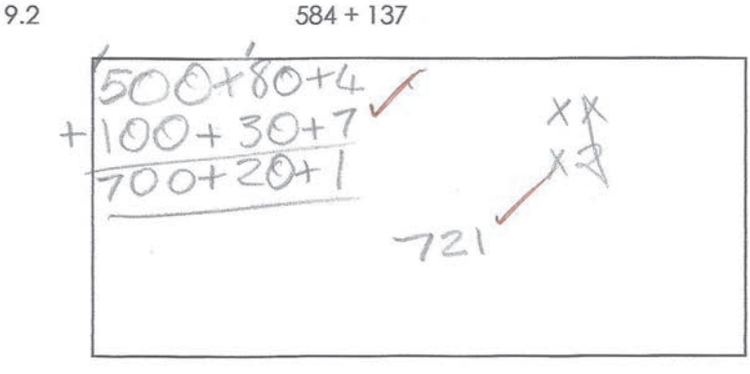
Item 9.1

- If the learner subtracted 213 from 795 and gave the answer 582: this shows the learner can identify the value of each of the digits in the given numbers and can use this knowledge to solve the problem;
- The learner should show the steps involved in solving the problem as well as provide the final answer.



Item 9.2

- If the learner added 137 to 584 and got the answer 721: this shows the learner can identify the value of the hundreds, tens and ones in each number and can use this knowledge to solve the problem;
- The learner should show the steps involved in solving the problem as well as provide the final answer.

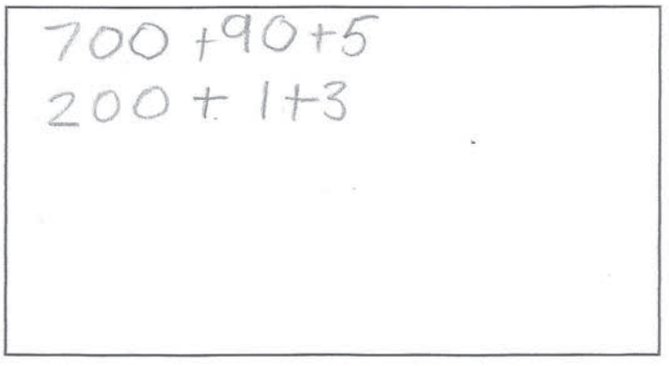


What would show evidence of partial understanding?

Item 9.1

- The response that follows shows partial understanding:
  - This learner was able to identify the value of each of the digits in the given numbers, but was then unsure of what to do next.
  - The learner did not know that the smaller number needed to be subtracted from the larger number in order to reach a final answer of 582.

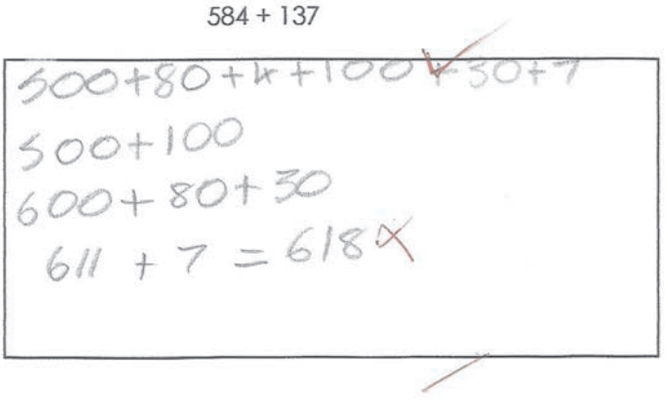
9.1 795 – 213



Item 9.2

- The learner who wrote the response below showed partial understanding.
  - This learner was able to identify the value of each of the digits in the given numbers and also knew that these numbers needed to be added together.
  - However, the learner was unsure of how to add 80 and 30 together.
  - The learner added 8 + 3 to get an answer of 11, rather than 80 plus 30 to get 110. This resulted in a final answer of 618 instead of 721.

9.2 584 + 137

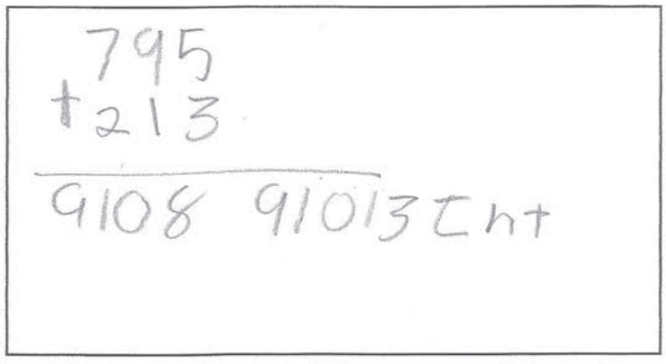


What would show evidence of no understanding?

Item 9.1

- The learner who wrote the response below showed no understanding of what was required for this question.
  - This learner added the two numbers together, rather than subtracting them.
  - The learner also used the vertical method to solve the problem instead of the breaking down method, but did not understand how to use the vertical method correctly
  - The learner did not understand how to add 90 + 10, but added 9 + 1 to get an answer of 10 instead and then wrote the 10 as part of the solution.

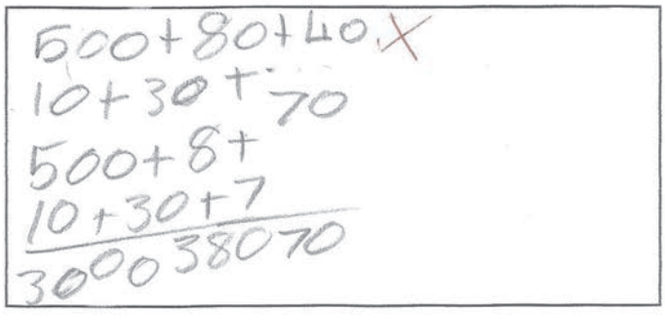
9.1 795 – 213



Item 9.2

- The learner who wrote the response below showed no understanding.
  - The learner was unable to break down the given numbers correctly.
  - The learner broke down 584 into 500 + 80 + 40 and 137 into 10 + 30 + 70.
  - This shows a lack of understanding of place value and that learner is thus unable to identify the value of each of the digits in a given number.
  - In addition to this, the learner did not know how to add the numbers. The numbers were repeated and laid out fairly haphazardly so it is unclear what was being added together to get an answer of 300038070.

9.2 584 + 137



What do the item statistics tell us?

Item 9.1

47% of learners answered the question correctly.

Item 9.2

37% of learners answered the question correctly.

Factors contributing to the difficulty of the item

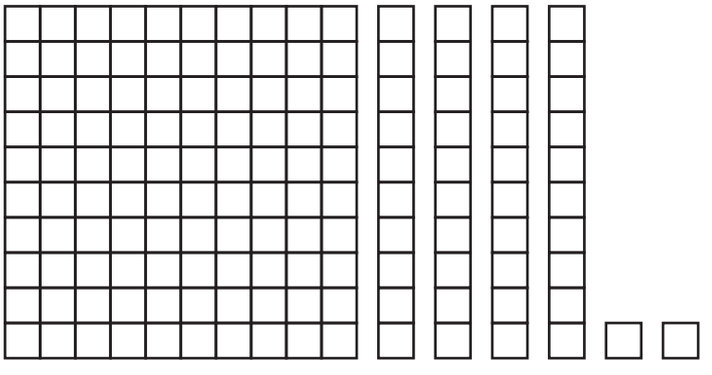
- These items may have been made more difficult by learners struggling to identify the value of each of the 3 digits in the given numbers.

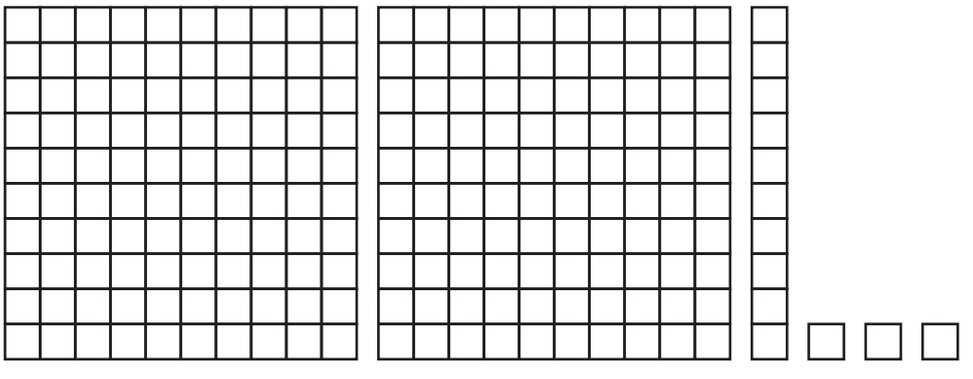
- Some learners may not have been able to calculate with three digit numbers, rather needing to operate at a lower number range.
- Learners may also have been unclear on the difference between subtraction and addition and have been unsure as to which operation to use for each item.
- Learners may not understand the breaking down method. If learners did not know how to follow the steps for this method of calculation, then they may have selected another method as an alternative (such as the vertical method).

Teaching strategies

Addition using base ten blocks

- First work on examples where no exchange between places is needed. The example given here is such an example. This helps establish the routine and the use of the aids to do the algorithm.
- Ask the learners to work in pairs or groups of four.
- Provide each pair or group of four with scrap paper/white boards and paper base ten blocks **(see printables).** The blocks should be cut up to show the ones, the tens and the hundreds. These paper blocks can be kept in envelopes, bank bags or empty margarine tubs.
- Write an addition problem on the board, for example: 142 + 213 = □
- Ask the learners to use the base ten blocks to make each number.





- Ask the learners to put the hundreds squares together and to count how many they have in total.
  - Say to the learners “Tell me about what you did”. Learners should respond “I added 1 hundreds square and 2 hundreds squares and I got 3 hundreds squares altogether”.

- As learners verbalise what they did, write the number sentence on the board: 100 + 200 = 300.

- Ask the learners to put the tens rows together and to count how many they have in total.
  - Say to the learners "Tell me about what you did". Learners should respond "I added 4 tens rows and 1 tens row and I got 5 tens rows altogether".
  - As learners verbalise what they did, write the number sentence on the board: 40 + 10 = 50.
- Ask the learners to put the ones blocks together and to count how many they have in total.
  - Say to the learners "Tell me about what you did". Learners should respond "I added 2 ones blocks and 3 ones blocks and I got 5 ones blocks altogether".
  - As learners verbalise what they did, write the number sentence on the board: 2 + 3 = 5.
- Circle the answers written on the board as shown:

$$100 + 200 = 300$$
$$40 + 10 = 50$$
$$2 + 3 = 5$$

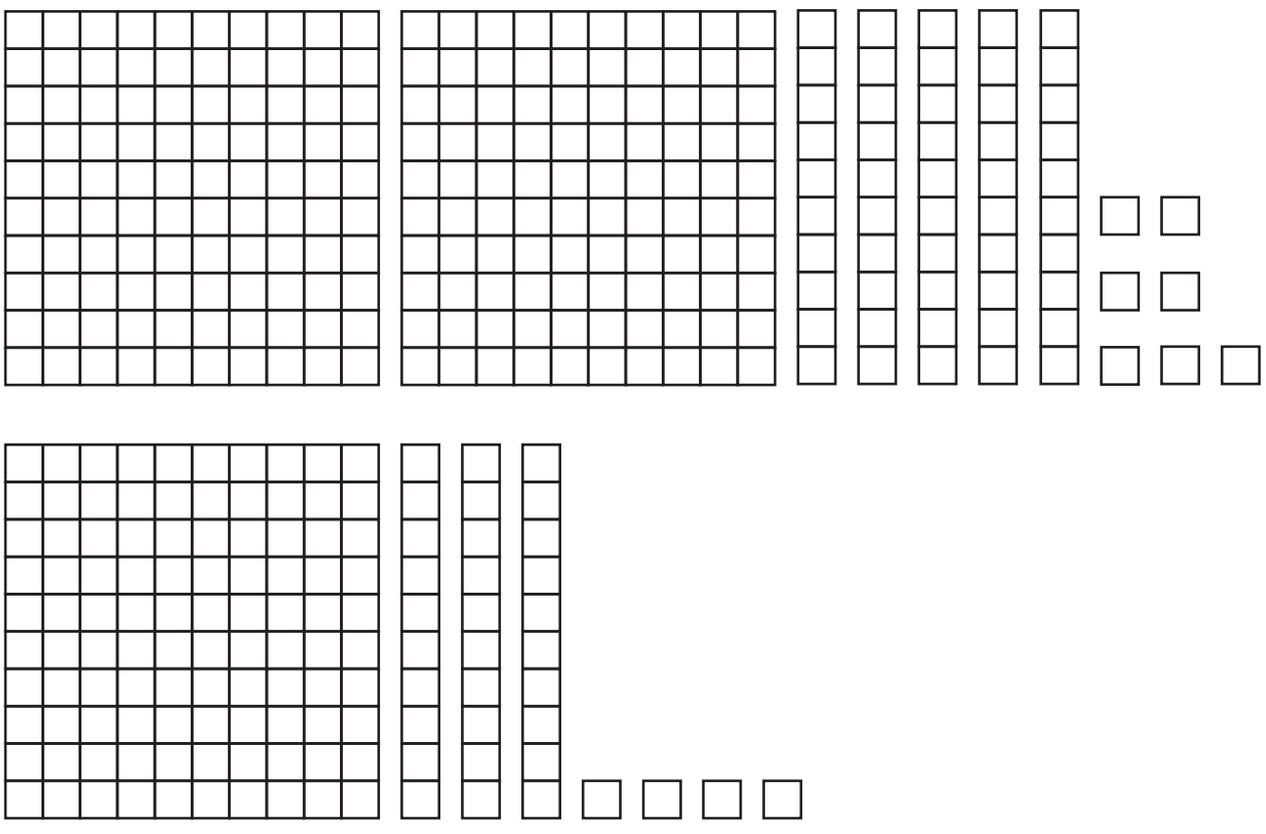
- Ask the learners to count their hundreds, tens and ones blocks and to write on their scrap paper/white boards the number that they have. Learners should write 355.
- Ask the learners to look at the number sentences on the board. Then ask the learners what they think they need to do with the circled numbers. Encourage the learners to realise that they need to add the circled numbers in order to get the answer to the original number sentence:

$$300 + 50 + 5 = 355$$

- Ask the learners if the answer to the number sentence is the same as the number they wrote on their scrap paper/white boards. Learners should realise that the counting of the base ten blocks gave them the same answer as they got from the number sentences.
- Do many examples where there is no need to exchange between places.
- Then you need to move on to examples where learners will get a total which involves an exchange, as in the ANA example, 584 + 137
- When learners lay out the blocks for 584 + 137 they will find:
  - In the units place: 4 + 7 = 11: they will have to exchange ten units for one ten.
  - In the tens place they will then have: 80 + 30 = 110, plus the 10 from the units = 120 and they will have to exchange ten tens for one hundred.
  - In the hundreds place they will then have 500 + 100 = 600, plus the 100 from adding the tens = 700
- The final answer should be 700 + 20 + 1 = 721.
- Remember to allow learners to work with the blocks to see how the exchange works.

Subtraction using base ten blocks

- Ask the learners to work in pairs or groups of four.
- Provide each pair or group of four with scrap paper/white boards and cut up paper base ten blocks **(see printables)**.
- Write a subtraction problem on the board, for example: 257 + 134 = □



- Ask the learners to use the base ten blocks to make each number.
  - Ask the learners "How many hundreds are there in the second number, 134?" Learners should answer "1 hundred".
  - Ask the learners to take 1 hundred away from the first number, 257. Learners should take away one hundreds square from the first number.
  - Ask the learners "How many hundreds squares are left over?" Learners should respond "1 hundred".
  - Write on the board 200 – 100 = 100.
  - Ask the learners, "How many tens are there in the second number, 134?" Learners should respond, "3 tens".
  - Ask the learners to take 3 tens away from the first number, 257. Learners should take away 3 tens rows from the first number.
  - Ask the learners, "How many tens rows are left over?" Learners should respond, "2 tens".
  - Write on the board: 50 – 30 = 20.
  - Ask the learners, "How many ones are there in the second number, 134?" Learners should

respond, “4 ones”.

- Ask the learners to take 4 ones away from the first number (257). Learners will take away 4 ones blocks from the first number.
- Ask the learners, “How many ones are left over?” Learners should respond, “3”.
- Write on the board 7 – 4 = 3.

- Circle the answers written on the board as shown:

200 - 100 = 100
50 - 30 = 20
- 4 = 3
7

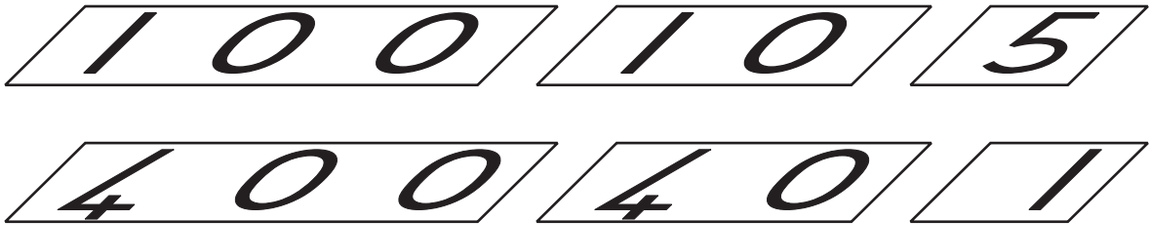
- Ask the learners to count the hundreds, tens and ones blocks left over and to write on their scrap paper/white boards the number that they have. Learners should write 123.
- Ask the learners to look at the number sentences on the board. Then ask the learners what they think they need to do with the circled numbers. Encourage the learners to realise that they need to add the circled numbers in order to get the answer to the original number sentence.

100 + 20 + 3 = 123

- Ask the learners if the answer to the number sentence is the same as the number they wrote on their scrap paper/white boards. Learners should realise that the counting of the base ten blocks gave them the same answer as they got from the number sentences.

Addition using place value cards

- Ask the learners to work in pairs or groups of four.
- Provide each pair or group of four with scrap paper/white boards and cut up place value cards **(see printables)**.
- Write an addition problem on the board, for example: 115 + 441 = □
- Ask the learners to use the place value cards to make each number.



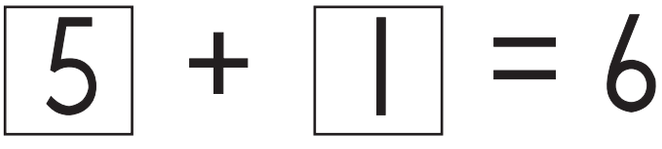
- Ask the learners to place the hundreds cards next to each other, with the one from the first number on the left and the one from the second number on the right.
- Ask the learners to put an addition sign in between the two cards, and to complete the number sentence.

- Say to the learners, “Tell me about what you did”. Learners should respond “I added 1 hundred and 4 hundreds and I got 5 hundreds altogether”.
- As learners verbalise what they did, write the number sentence on the board: 100 + 400 = 500.

- Ask the learners to place the tens cards next to each other, with the one from the first number on the left and the one from the second number on the right.
- Ask the learners to put an addition sign in between the two cards, and to complete the number sentence.

- Say to the learners “Tell me about what you did”. Learners should respond “I added 1 ten and 4 tens and I got 5 tens altogether”.
- As learners verbalise what they did, write the number sentence on the board: 10 + 40 = 50.

- Ask the learners to place the ones cards next to each other, with the one from the first number on the left and the one from the second number on the right.
- Ask the learners to put an addition sign in between the two cards, and to complete the number sentence.

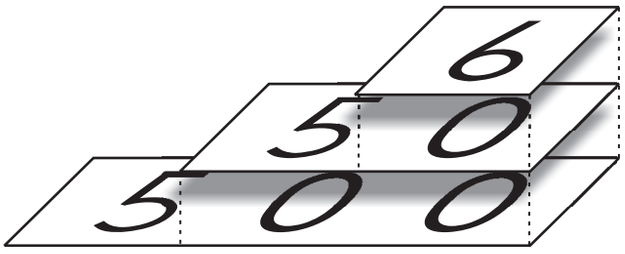


- Say to the learners “Tell me about what you did”. Learners should respond “I added 5 ones and 1 one and I got 6 ones altogether”.
- As learners verbalise what they did, write the number sentence on the board: 5 + 1 = 6.

- Circle the answers written on the board as shown:

100 + 400 = 500
10 + 40 = 50
5 + 1 = 6

- Ask the learners to find each of the circled numbers on their place value cards and to make the number:



- Ask the learners to add the numbers together and to write the number sentence:

$$500 + 50 + 6 = 556$$

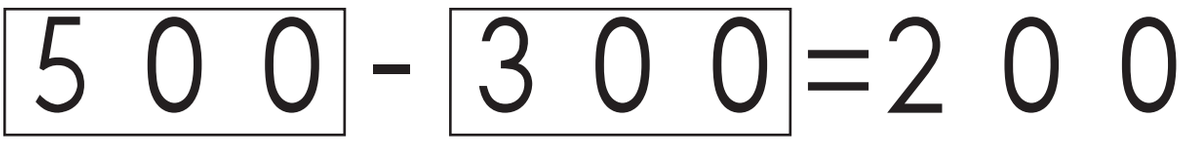
- Remember to do examples that do not need exchanges (or regrouping with the base ten cards) and ALSO examples where exchanges are needed.
- Show all of the working using the cards when you demonstrate.

Subtraction using place value cards

- Ask the learners to work in pairs or groups of four.
- Provide each pair or group of four with scrap paper/white boards and cut up place value cards **(see printables for item 1)**.
- Write a subtraction problem on the board, for example: 569 - 325 = □
- Ask the learners to use the place value cards to make each number.

- Ask the learners to place the hundreds cards next to each other, with the one from the first number on the left and the one from the second number on the right.

- Ask the learners to put a subtraction sign in between the two cards and to complete the number sentence.



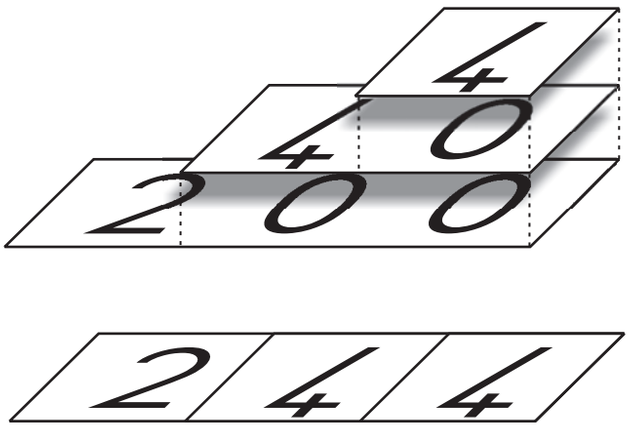
  - Write on the board: 500 – 300 = 200.
- Ask the learners to place the tens cards next to each other, with the one from the first number on the left and the one from the second number on the right.
- Ask the learners to put a subtraction sign in between the two cards and to complete the number sentence.

  - Write on the board: 60 – 20 = 40.
- Ask the learners to place the ones cards next to each other, with the one from the first number on the left and the one from the second number on the right.
- Ask the learners to put a subtraction sign in between the two cards and to complete the number sentence.

  - Write on the board: 9 – 5 = 4.
- Circle the answers written on the board as shown:

500 - 300 = 200
60 - 20 = 40
9 - 5 = 4

- Ask the learners to find each of the circled numbers on their place value cards and to make the number:



- Ask the learners to add the numbers together and to write the number sentence:

$$200 + 40 + 4 = 244$$

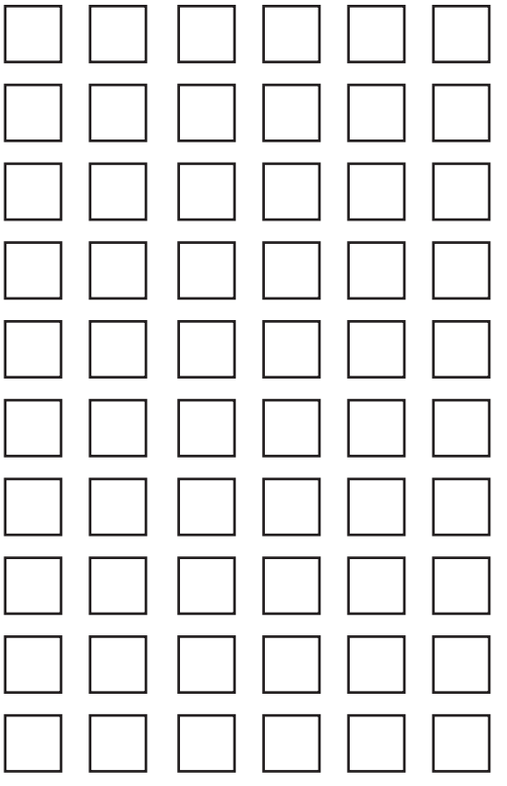
Another example of how breaking-down as calculation strategy can be tested

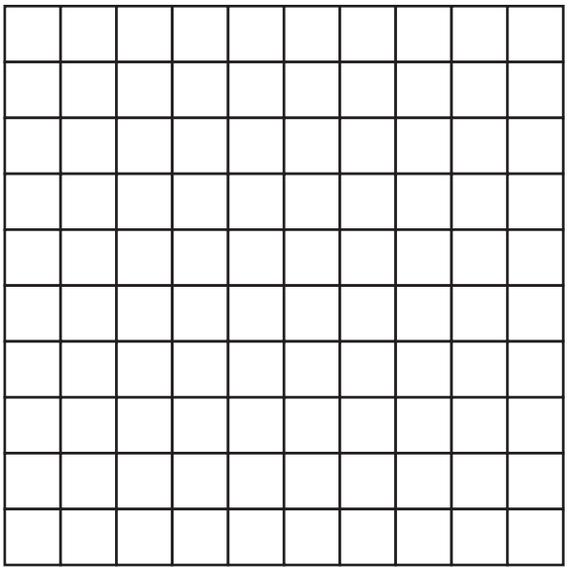
ANA 2014 Grade 3 Mathematics Item 22

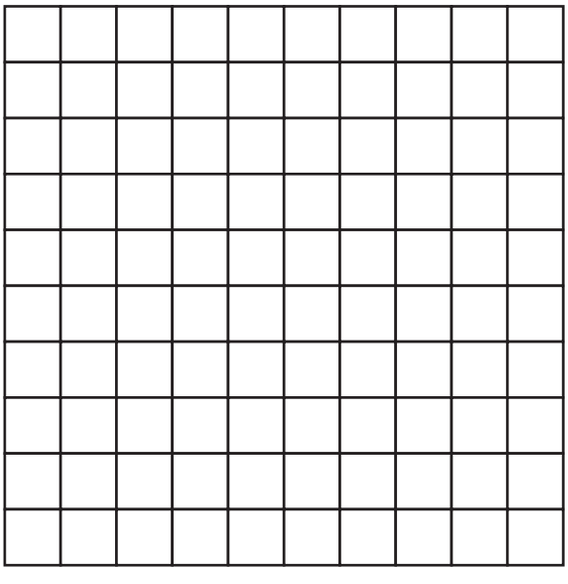
22. Calculate 489 - 256 by using the 'breaking down' method.

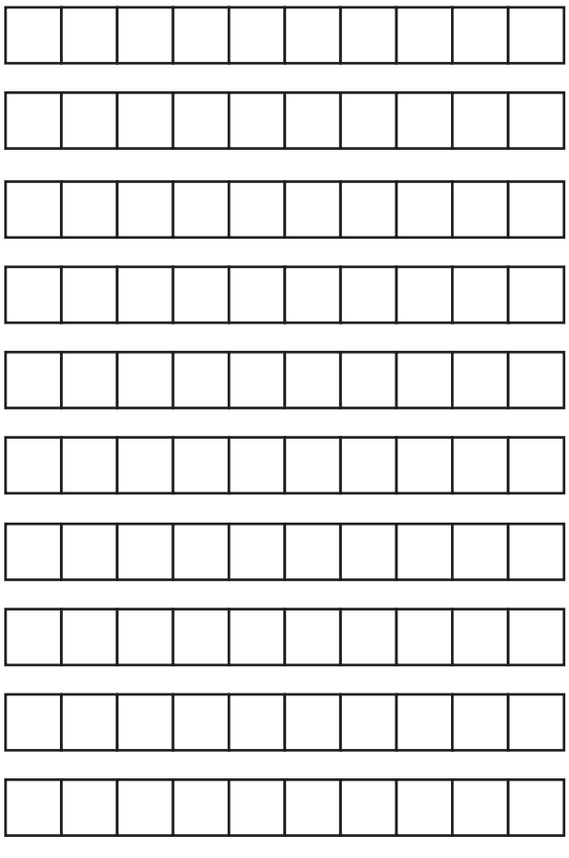
Notes:

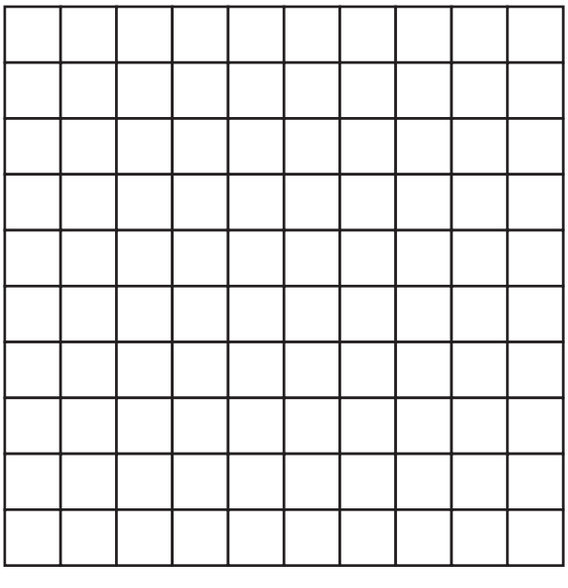
Printable: Base ten blocks







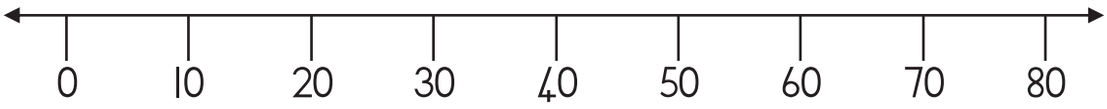




Addition using a number line

ANA 2013 Grade 3 Mathematics Item 25

25. Show how you will use the number line to calculate 20 + 30.



What should a learner know to answer this question correctly?

Learners should be able to:

- Locate numbers on a number line;
- Show their calculation by drawing hops on the number line.

Where is this topic located in the curriculum? Grade 2 Term 3

Content area: Numbers, Operations and Relationships (Context-free calculations).

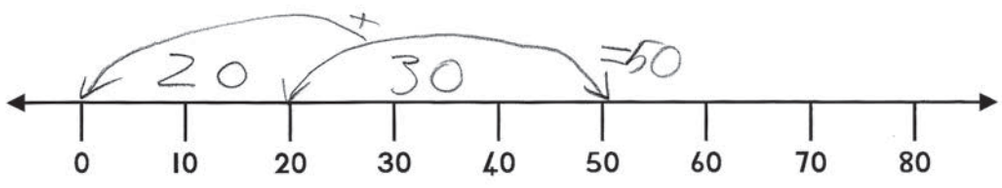
Topic: Techniques (methods or strategies).

Concepts and skills:

- Use the following techniques when performing calculations: number lines.

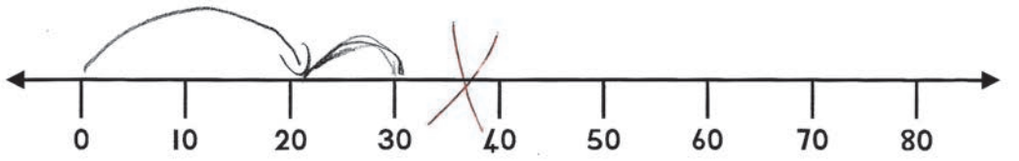
What would show evidence of full understanding?

- If the learner started at zero and made a jump to the number 20 and then another jump from 20 to 50;
- Alternatively, if the learner made a small jump from 0 to 10, followed by another one from 10 to 20 and then three small jumps from 20 to 30, 30 to 40 and 40 to 50;
- Both these methods show that the learner is aware of where 20 is situated on the number line and that the learner knows 30 must be added to 20, thereby getting an answer of 50.

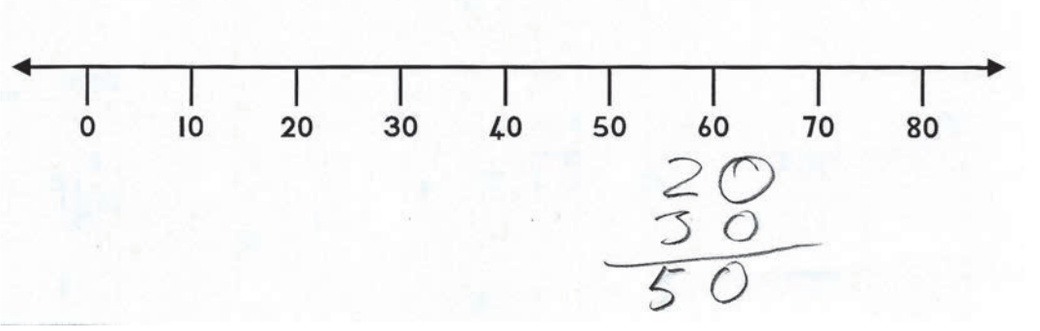


What would show evidence of partial understanding?

- In the example below, the learner placed 20 correctly on the number line;
- However, the learner then became confused about how to add 30 to 20;
- The learner gave 30 as the final answer instead of adding 30 to 20 to get the answer 50.

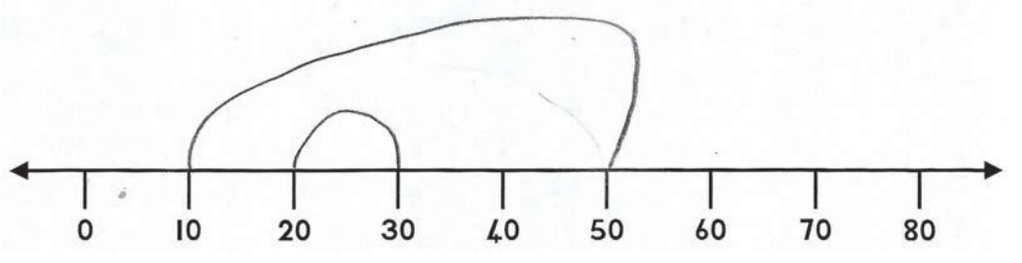


- The learner who gave the response illustrated below was able to show the addition of 20 and 30, but did not use a number line as required by the question;
- The learner chose instead to solve the problem through use of the vertical method;
- This suggests that the learner does not understand how to add using the number line. (The curriculum specifies certain methods of calculation and learners must be shown how to use those methods so that if they are required to use them, as in this ANA item, they are able to do so).



What would show evidence of no understanding?

- If the learner did not give an answer and drew a response such as the one that follows;
- This learner was unsure of what was expected and drew two unconnected jumps on the number line;
- The small jump from the 20 to the 30 may have been drawn in recognition of the two numbers specified in the question, but there is no understanding shown of the need to add these two numbers;
- Even though the learner's second bigger jump landed on the 50, there is no indication that the learner realised that 50 is the answer to 20 + 30, because this jump started at 10, which does not appear in the problem.



What do the item statistics tell us?

32% of learners answered the question correctly.

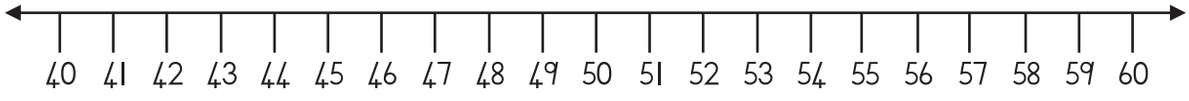
Factors contributing to the difficulty of the item

- Learners may have found this item more difficult because the number 30 follows 20 when counting in tens. Learners may not have realised that they needed to add 30 more and instead just jumped from the 20 to the 30 because these are consecutive numbers.
- The difficulty of this item may be linked to the methods used to teach addition. Learners may have been taught to solve addition problems in a specific manner and may perhaps be unfamiliar with this method of solution. Learners may then have guessed which jumps to show on the number line, rather than understanding the use of the number line to solve the problem.

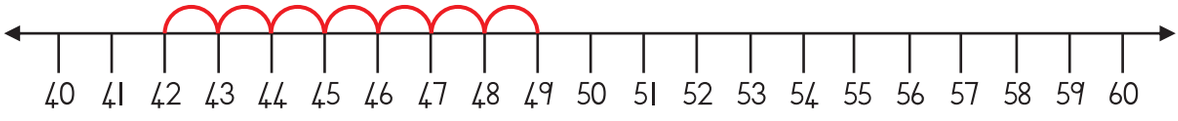
Teaching strategies

Counting on a number line

- Ask the learners to work in pairs or groups of four.
- Give each pair or group of four a number line and some counters.
- This example uses a number line labelled in ones from 40 up to 60.



- Ask learners to find the number 42 on the number line.
- Ask learners "How did you know this was number 42?" Learners may respond by saying "I know that the number 42 has a 4 and a 2 in it" or "I know that the number 42 comes 2 after the number 40, etc."
- Ask learners to put a counter on the number 42.
  - Ask learners to count 7 places from 42, using their fingers to show the jumps on the number line.



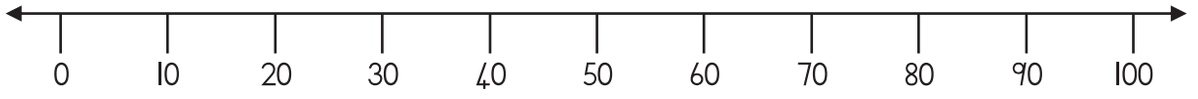
- Ask learners "Which number did you start counting from?" Learners may respond "42" or '43".
  - If learners responded 42, then explain that, because they used the counter to cover up that number, they cannot say the number 42: they need instead to move onto the next number to the right, 43.
- Learners need much practice in counting on from a given number, so provide them with a variety of number lines or ask them to draw number lines on scrap paper or white boards. The number lines can vary in number ranges and can also increase by differing amounts, e.g. ones, fives, tens, hundreds, etc.
- After learners have become comfortable with counting by using their fingers to show their jumps on

a number line, ask the learners to draw the jumps on the number line with their pencils.

- Ask learners to "Show on your number lines how you count on 8 places from 52".

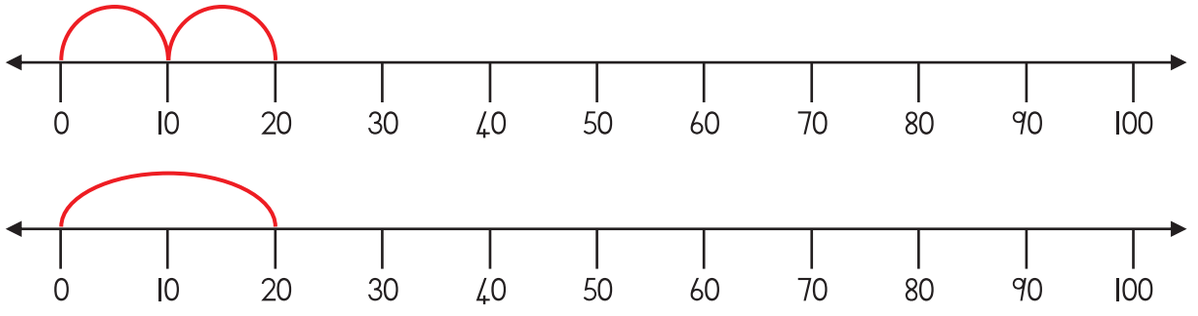
Adding on a number line

- Using the previous activity as a starting point, ask the learners to work in pairs or groups of four.
- Give the pairs or groups of four a number line with the multiples of 10 from 0 to 100.
- Write a problem on the board: 50 + 30 = □.

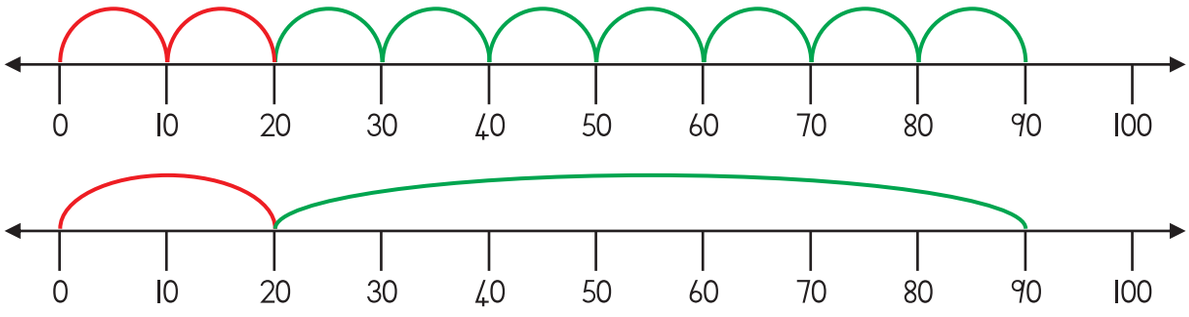


  - Ask learners "Where do you think you need to put your finger to start with on the number line?" Learners are likely to say "On the zero" or "On the fifty".
  - Ask learners to put their fingers on the zero and to then count in tens until they get to 50.
  - Ask learners to look at the number sentence and ask "What do you think you need to do next?" Learners should respond "We need to move our fingers to the 30" or "We need to add 30".
  - Explain to the learners that, as they did in the previous activity, they are going to count on 30 from 50.
  - Ask learners to jump with their fingers as they count in tens from 50. Learners should say "10, 20, 30" as they place their fingers on 60, 70 and 80.
  - Ask learners which number their fingers are on. Learners should respond "80".
  - Ask learners, "So what is 50 + 30?" Learners will realise that 50 + 30 = 80.

- When learners have had much practice with this, using their fingers to show their jumps, you can move on to asking learners to use a pencil to mark their jumps on the number line.
- Write the number sentence 20 + 70 = on the board.
  - Ask learners to draw jumps on their number lines with their pencils to show how they would get to 20.
  - Say to the learners "Tell me about the jumps that you drew". Learners should respond "I drew two jumps – one from 0 to 10 and then one from 10 to 20" or "I drew one big jump from 0 to 20".



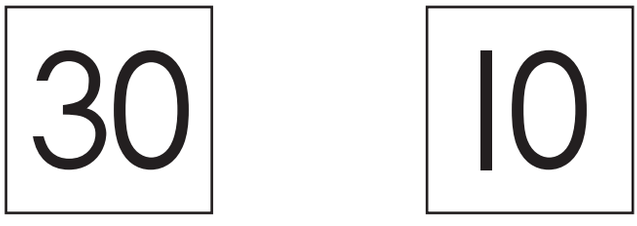
- Discuss with the learners the difference between these two versions of recording. Tell the learners that the two smaller jumps show how the learners counted in tens to get to 20, whereas the one big jump shows how learners went straight from the zero to the first number of the number sentence.
- Ask learners "What are you going to do next?" Learners should respond that they will count in tens from 20.
- Ask learners to draw their jumps on their number lines with their pencils.
- Say to the learners "Tell me about the jumps that you drew". Learners should respond "I drew seven jumps – from 20 to 30, 30 to 40, 40 to 50, 50 to 60, 60 to 70, 70 to 80 and from 80 to 90" or "I drew one big jump from 20 to 90".



- Discuss with the learners the difference between these two versions of recording. Tell the learners that the seven smaller jumps show how the learners counted in tens to get to 90, whereas the one big jump shows the learners went straight from 20 to 90.
- Ask learners which number their pencil jumps ended on. Learners should respond "90".
- Ask learners "So what is 20 + 70?" Learners will realise that 20 + 70 = 90.

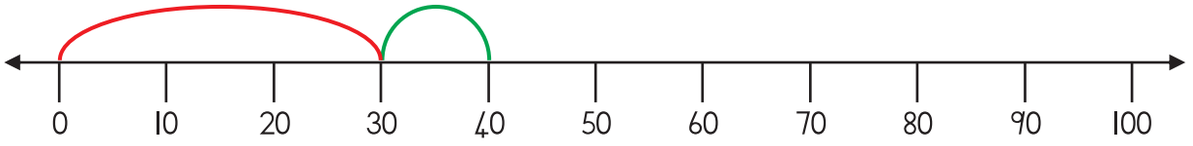
Addition game

- Ask learners to work in pairs or groups of four.
- Provide each group with a set of tens cards which have been cut out **(see printables)** and scrap paper or a white board.
- Provide each learner with a 0 to 100 number line which shows the multiples of 10.
  - Ask learners to turn the tens cards face down on the desks.
  - Then ask one learner in each group to turn over two cards, for example:



  - Learners must then discuss what they think the number sentence should be and then write the sentence on their scrap paper or white boards, for example: 30 + 10 = □.

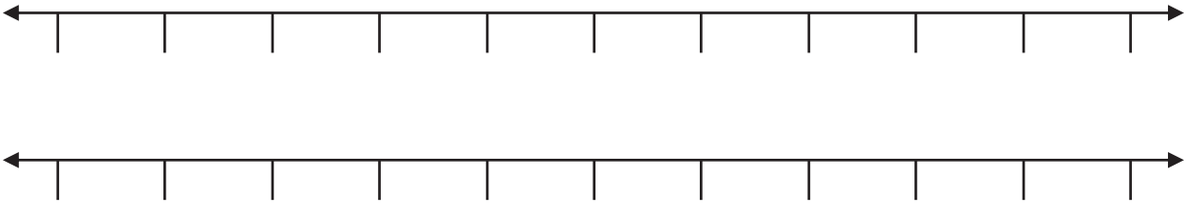
- Learners must then draw jumps on their number lines to show their calculations.



- Learners must then insert the answer into the number sentence written on the scrap paper or white board, for example: 30 + 10 = 40
- The learners must then compare their number lines and discuss how they solved the problem.
- Turn the tens cards face down on the desk again and shuffle them around.
- Ask a second learner in each group to turn over two cards and the learners should follow the above steps again.
- As the groups will be working independently, you will be free to move around the groups and assist learners as needed. Alternatively you can work with a small group of learners in a teacher guided activity to remediate misconceptions.

Blank number lines

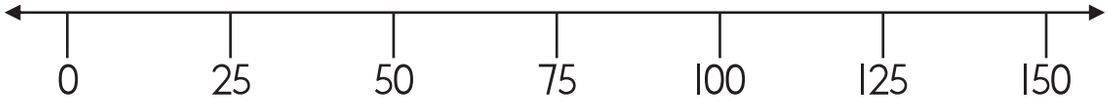
- Ask learners to work in groups of four.
- Give each learner a sheet of blank number lines.
- Ask one learner from each group to suggest a number range for the first number line on the page. The number range will depend on the learner's own ability.
  - All the learners in the group will fill in the numbers on the first number line using the range suggested.
  - The learner who suggested the number range will then suggest an addition problem for the group to solve using that number line.
  - All the group members will solve the addition problem by drawing the hops onto their number lines.
  - The group members can then discuss their solutions amongst themselves.
  - Following this, another group member can then suggest a new number range and addition problem for the group (as per the steps above).
  - This can continue until all the number lines have been completed.
- Note: all the group members will do all the addition problems, so that no learner is left sitting observing whilst the rest of the group solves a problem.



Another example of how addition on a number line can be tested

ANA 2014 Grade 3 Mathematics Item 28

28. Draw jumps on the number line to show that 25 + 25 = 50.



Notes:

Printable: Number cards

10	60
20	70
30	80
40	90
50	100

Repeated addition leading to multiplication

ANA 2013 Grade 3 Mathematics Item 14

14. Complete:

4 + 4 + 4 + 4 + 4 + 4 = _____ × 4

What should a learner know to answer this question correctly?

Learners should be able to:

- Group items and count their groupings;
- Recognise the repeated addition pattern;
- Understand the meaning of the equal sign in mathematics;
- Understand that repeated addition is also a multiplication operation;
- Understand that in repeated addition the same number is repeatedly added.

Where is this topic located in the curriculum? Grade 3 Term 2

Content area: Number, Operations and Relationships.

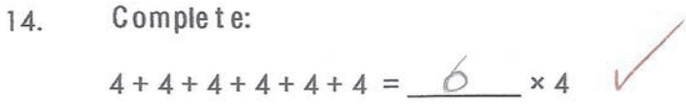
Topic: Repeated addition leading to multiplication.

Concepts and skills:

- Multiply numbers 1 to 10 by 2, 3, 4, 5, 10 to a total of 99;
- Use appropriate symbols (x, =, ☐,=).

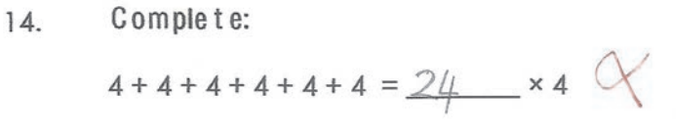
What would show evidence of full understanding?

- To show evidence of full understanding the learner ought to express the repeated addition sum using multiplication correctly.



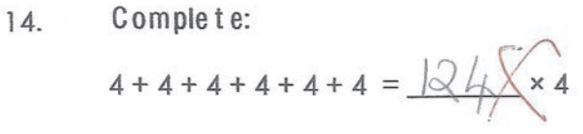
What would show evidence of partial understanding?

- If the learner gave the sum of the repeated addition numbers as the answer, this shows that the learner does not understand the meaning of the equal sign. This is seen in the following learner response, where the answer 24 is given instead of the answer 6 which is the factor needed to balance the two sides of the equation.



What would show evidence of no understanding?

- If the learner gave the incorrect answer to the question; or
- If the learner made no attempt to complete the number sentence.



What do the item statistics tell us?

27% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Learners may not understand the correct mathematical meaning of an equal sign;
- Learners may not have fully mastered grouping and multiplication.

Teaching strategies

Understanding the meaning of an equal sign

- Learners must understand that an equal sign does not just mean "the answer".
- The symbol = shows that what is on the left of the sign is equal in value or amount to what is on
- the right of the sign.

Examples: $5 + 4 = 9$ means that $5 + 4$ is equal to 9.
60 seconds = 1 minute means that 60 seconds is equal to 1 minute.

Grouping of concrete items:

- Learners need to understand the meaning of an equal sign.
- Explain the meaning to them, and give examples of how to use an equal sign.

  Examples: $5 = 5$

  $5 = 2 + 3$

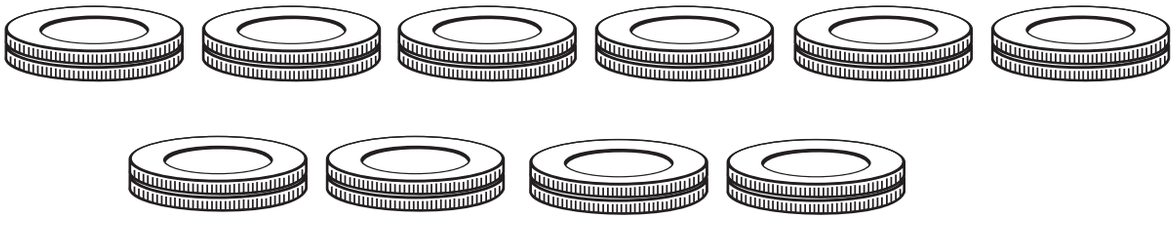
  $5 + 5 = 10$

  $5 + 5 + 5 + 5 + 5 + 5 = 6 \times 5$

- After you have explained this to your class you could work through some of the following activities.
- Learners must be able to use concrete objects to demonstrate repeated addition of groups of items, do repeated addition and apply multiplication.
- Allow learners to do many such activities to allow them to consolidate the idea that repeated addition leads to multiplication.

- Give each learner (or group of learners) 30 counters or Unifix cubes.*
  - Ask learners put counters/Unifix into groups of 2, 3, 4 and/or 5.
  - Ask learners to count in multiples of 2, 3, 4 and 5, referring to their counters.
  - Lead learners to realise that counting in multiples is the same as adding the same number repeatedly.
  - It is very important that learners write the number sentences. They must not just do practical work and oral counting. They need to practice writing the mathematical number sentences that express the work they have done.

- Ask learners to put 20 counters into groups of 2



- Ask the questions:
  - How many groups? 10
  - How many in each group? 2
  - How many counters altogether? 20
  - How do I write this using numbers and symbols?

$2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 20,$

10 groups of $2 = 20$, or $10 \times 2 = 20$

- Ask learners to put 30 counters into groups of 3

- Ask the questions:
  - How many groups? 10
  - How many in each group? 3
  - How many counters altogether? 30
  - How do I write this using numbers and symbols?

    $3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 30,$
    10 groups of $3 = 30$, or $10 \times 3 = 30$

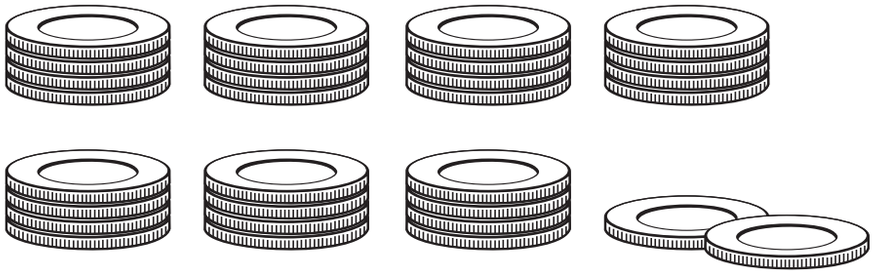
- Ask learners to put 30 counters into groups of 5

- Ask the questions:
  - How many groups? 6
  - How many in each group? 5
  - How many counters altogether? 30
  - How do I write this using numbers and symbols?

    $5 + 5 + 5 + 5 + 5 + 5 = 30,$
    6 groups of $5 = 30$, $6 \times 5 = 30$, or
    $5 + 5 + 5 + 5 + 5 + 5 = 6 \times 5$

  - Why are the number sentences equal? The two number sentences express the same number so they are equal to each other.

- Ask learners to put 30 counters into groups of 4

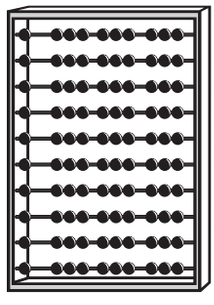


- Ask the questions:
  - How many groups? 7
  - How many in each group? 4
  - How many counters altogether? 30 but not all are grouped
  - How do I write this (the grouped part) using numbers and symbols?

    $4 + 4 + 4 + 4 + 4 + 4 + 4 = 28$,
    7 groups of $4 = 28$, $7 \times 4 = 28$, or
    $4 + 4 + 4 + 4 + 4 + 4 + 4 = 7 \times 4$

  - Encourage learners to observe that if you put 30 counters into groups of 4, you can only make 7 groups of 4 and two counters will be left over: they will not form another group.

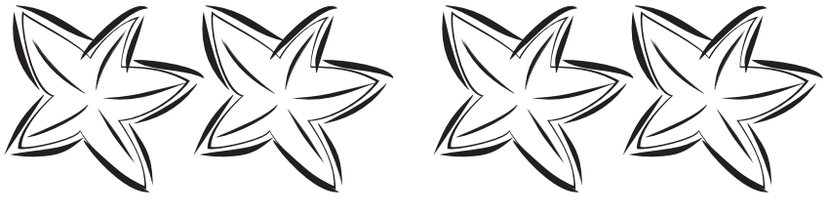
Repeated addition by grouping objects



- Ask learners to group counters on their abacuses and count in 2s, 3s, 4s and 5s.
- Ask learners to talk about the counting they have done and to write down equations of some of their counting to practice using the equal sign correctly,

  Examples: $3 + 3 + 3 = 3 \times 3$,
  $3 + 3 + 3 = 9$,
  $3 + 3 + 3 + 3 + 3 + 3 = 3 \times 6 = 18$ (etc.)

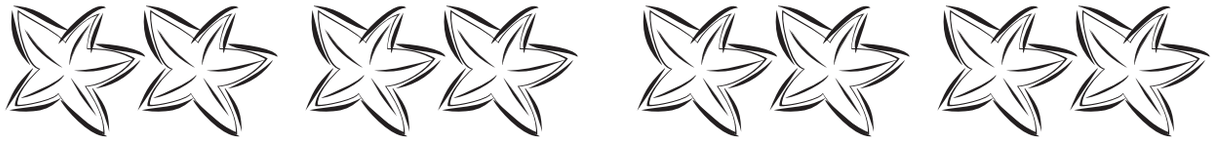
- Ask learners to make two groups of 2.



  - Ask if anyone can show you two different ways you can write this, i.e. $2 + 2 = 4$ or $2 \times 2 = 4$.
  - Discuss what the different signs mean and why both number sentences make 4.

- Ask learners to put out another two groups of 2.
  - Ask learners to count in twos, pointing to the groups.
  - Ask if anyone can write the sum in two different ways,

    i.e. 2 + 2 + 2 + 2 = 8 or 4 x 2 =8

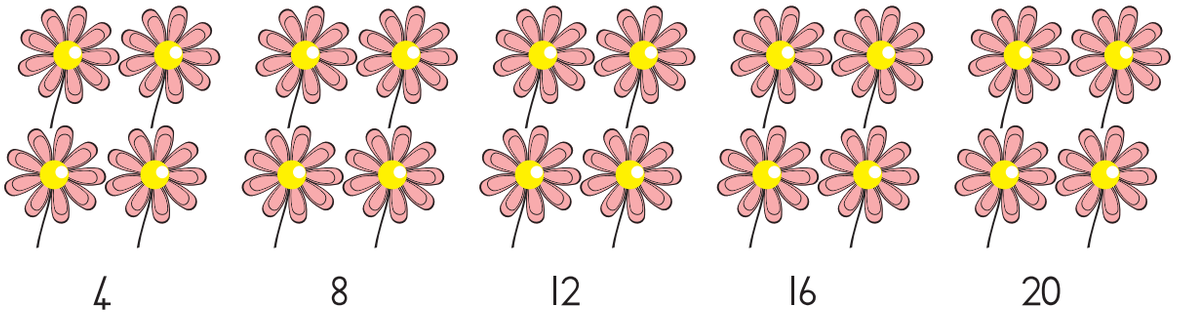


  - Discuss the meaning of both number sentences.
- Repeat this until learners have made ten groups of 2, each time writing the repeated addition as well as the multiplication.

Repeated addition by counting in multiples of 2, 3, 4, 5 and 10 between 0 and 100

- Always start with one digit numbers and proceed to two digit numbers, in order to help the learners to understand the concept of repeated addition that leads to multiplication;
- Ask learners to count in multiples of 2, 3, 4, 5, and 10 between 0 and 100;
- Encourage learners to focus on counting faster using strategies.

- Repeated addition of 4:
  - Learners can group counters in fours;
  - Pictorial representations can be used, for example groups of flowers:



- Give learners a 100 chart **(see printables)** and ask them to count in 4s.

- Give learners 3 empty/blank 100 charts **(see printables)** or use squared paper and help the learners to draw 10 x 10 squares on the paper.
- Ask learners to fill in the multiples of 3, 5 and 10 as they count.
  - Learners may use a different colour for each multiple they count in, e.g. multiples of 3 in green, of 5 in red, of 10 in yellow.
  - Encourage learners to observe which numbers appear in all three multiples.

Another example of how repeated addition leading to multiplication can be tested

ANA 2014 Grade 3 Mathematics Item 18.1

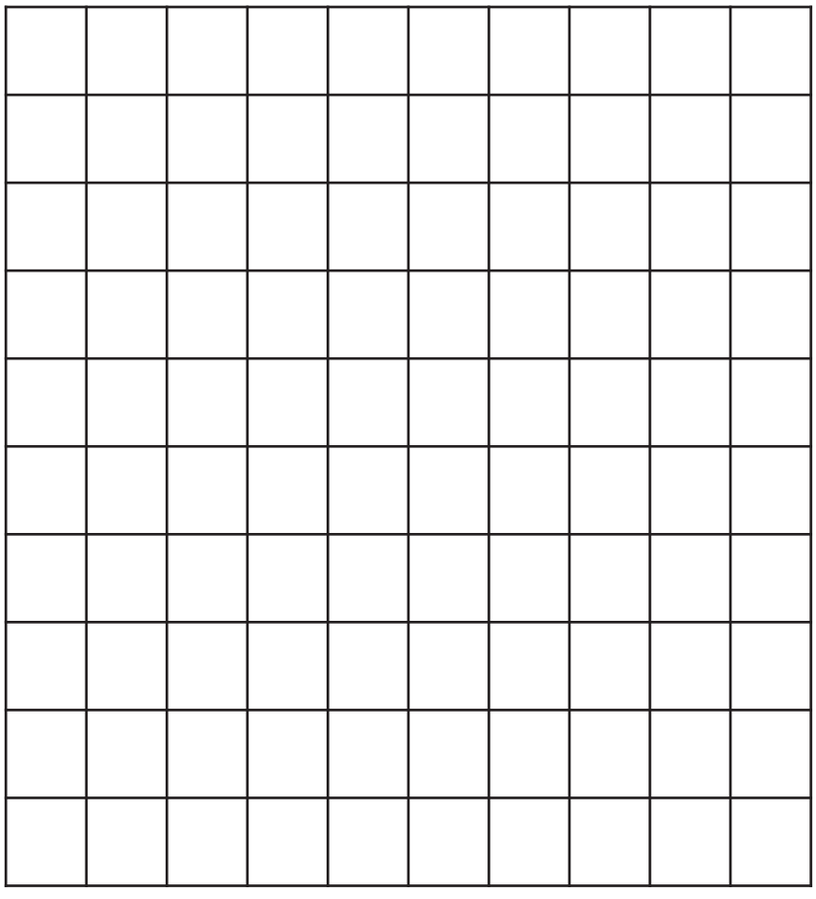
18.1	In the toy box there are 12 soccer balls, 12 rugby balls and 12 tennis balls. How many balls are there altogether?

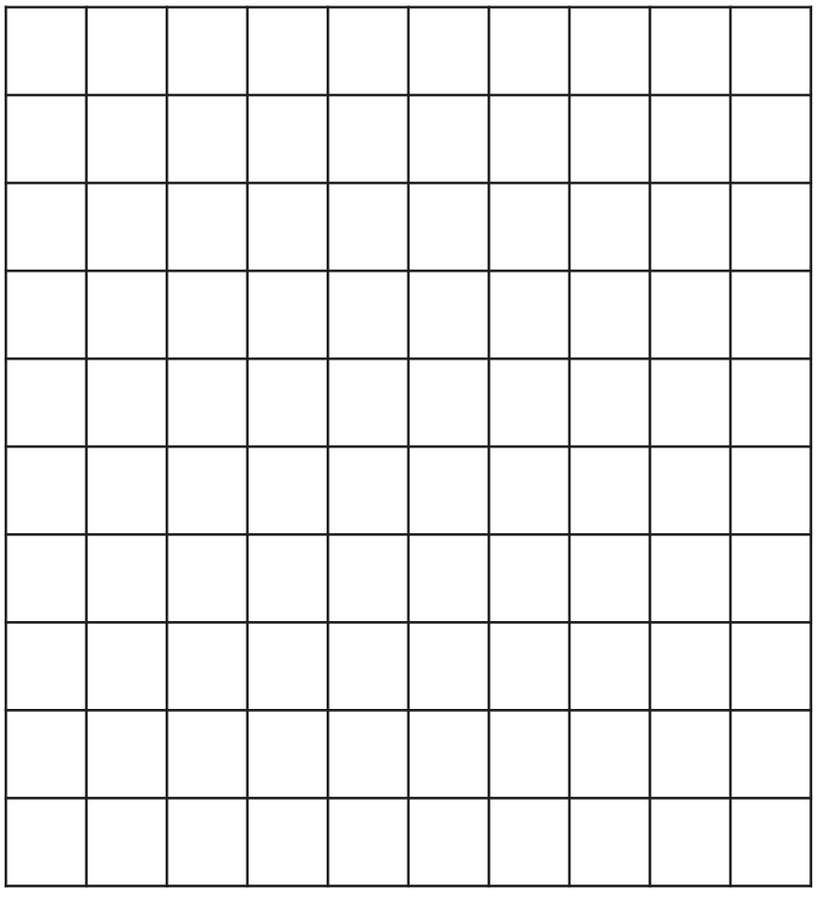
Notes:

Printable: 100 chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Printable: Blank 100 chart





Doubling and halving

ANA 2013 Grade 3 Mathematics Item 3

3. 39 doubled =

A 69
B 79
C 78
D 96

What should a learner know to answer this question correctly?

Learners should be able to:

- Understand the mathematical vocabulary relating to doubling of a number: doubled, doubling or double;
- Understand that when you double a number you need to add the same number twice or multiply the number by two;
- Carry tens and add them correctly;
- Build up and break down a number using tens and units.

Where is this topic located in the curriculum? Grade 3 Term 1

Content area: Numbers, Operations and Relationships.

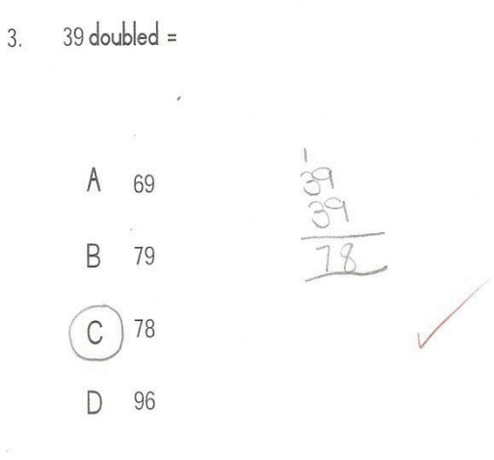
Topic: Problem solving techniques.

Concepts and skills:

- Use the following techniques when solving problems and explain solutions to problems: Doubling and halving.

What would show evidence of full understanding?

- Full understanding is shown if the learner indicated the correct answer is C, i.e.78. This shows that:
- The learner is able to add two (2) of the same numbers, i.e. 39 + 39 = 78;
- The learner is able to apply multiplication correctly, i.e. 2 x 39 = 78;
- The learner is able to break down numbers into tens and units to solve the problem;
- The learner is able to solve problems by carrying units to tens correctly.



What would show evidence of partial understanding?

- If the learner chose option A, this shows that the learner only doubled the tens and not the units. This indicates some understanding of doubling, but the learner has not fully mastered the skill of doubling as yet.

What would show evidence of no understanding?

- If the learner selected option B or D: this shows no understanding on the part of the learner, as answers B and D do not have any relation to the doubling of 39;
- If the learner did not answer the question;
- If the learner chose more than one answer as the correct response.

What do the item statistics tell us?

- 52 % of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Poor understanding of the concept and skills tested in this item, such as doubling;
- Incorrect calculation of two numbers when carrying tens;
- Multiple choice questions could be a contributing factor that made the item difficult as learners have to apply elimination skills when selecting the correct answer. Teachers very seldom give learners multiple choice questions as a basic teaching strategy.

Teaching strategies

Understanding doubling and halving through practical examples

Learners should:

- Use different techniques when solving problems involving doubling and halving;
- Know how to apply breaking down and building up to solve doubling and halving;
- Understand the mathematical terminology of doubling and halving.

Doubling:

- When you increase a number by adding the same number to it, e.g. 24 stones, add 24 more stones equals 48 stones.

Halving:

- When you equally share or divide a number into two groups of the same size, each half has the same amount, e.g. 24 stones shared equally among two learners is 12 stones each.
- The teacher should always introduce doubling and halving with smaller numbers to make it easier for learners to understand. Teaching doubling and halving simultaneously will assist learners to develop an understanding of the concepts and the vocabulary and applying the correct skills.
- Show learners practically what to do when they double or halve a number.

When you double a number:

You have 14 stars

Double the 14 stars = 28 stars

Repeated addition of the number: 14 + 14 = 28

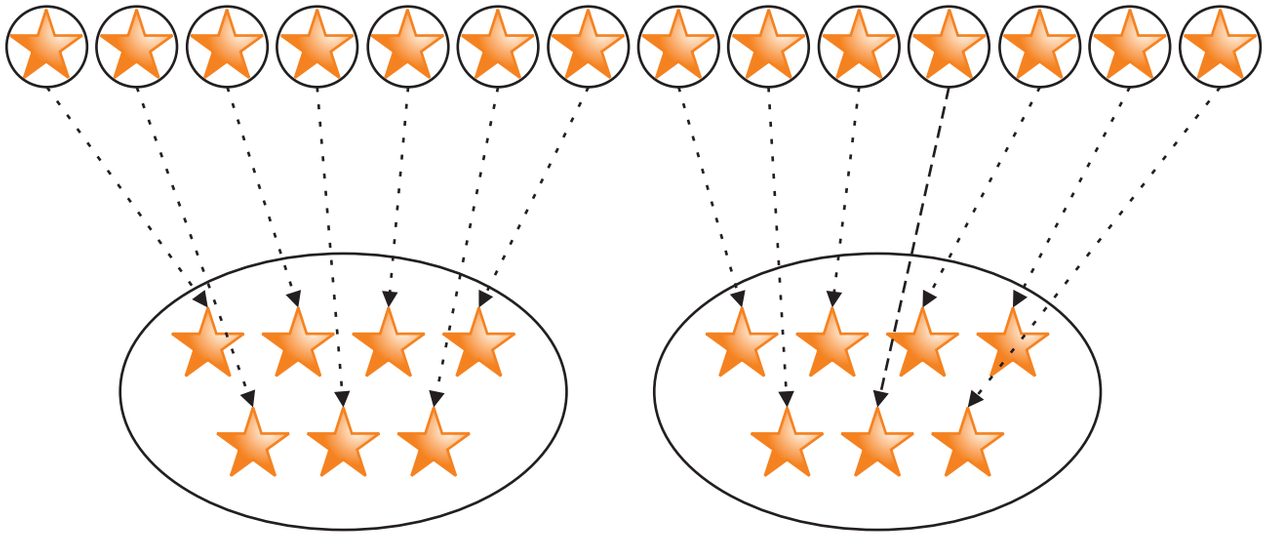
or

Double the number by the same number: 2 x 14 = 28

When you halve a number:

You have 14 stars.

Halve the stars = 7 stars



Share or divide the stars equally into two halves: 14 ÷ 2 = 7

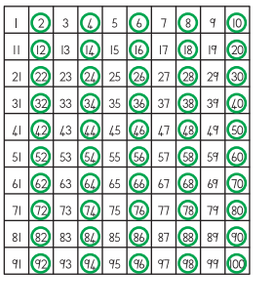
Numeric strategies for doubling and halving

- Use flash cards **(see printables)** to show a number and ask learners to double or halve the number shown.

1	2	3	4	13	14	15	16
5	6	7	8	17	18	19	20
9	10	11	12	21	22	23	24
37	38	39	40	49	50	51	52
41	42	43	44	53	54	55	56
45	46	47	48	57	58	59	60

- Play games and allow learners to double or halve the scores.
- Play with number dice and ask learners to double or halve the numbers thrown, e.g. half of 12 is 12 ÷ 2 = 6, double 12 is 12 + 12 = 24.

- Circle the even numbers on the 100 chart – an even number will have no remainders when you halve the number.
- NOTE: If you halve an odd number, the number will have a remainder. Learners should be able to divide the remainder. For example, half of 3 is 1 remainder 1. The remaining 1 can be divided between two learners so that each one will receive a half. This means that half of 3 is 1 and a half.



Doubling and halving using the building up and breaking down strategy

- Learners may decompose (build up and break down) a number in order to work with smaller numbers before doubling and/or halving a number.

Example: Doubling

240 doubled is:

(240) + (240)

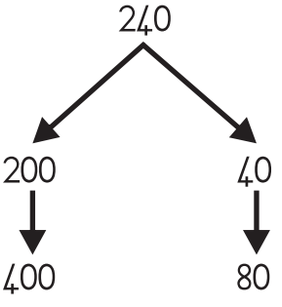
(200 + 200) + (40 +40)

(400) + (80)

480

or

240 doubled is:



480

480 is double 240

Example: Halving

36

30 6

15 3

18

18 is half of 36

or

36

(30 - 15) + (6-3)

15 + 3

10 + (5+3)

18 is half of 36

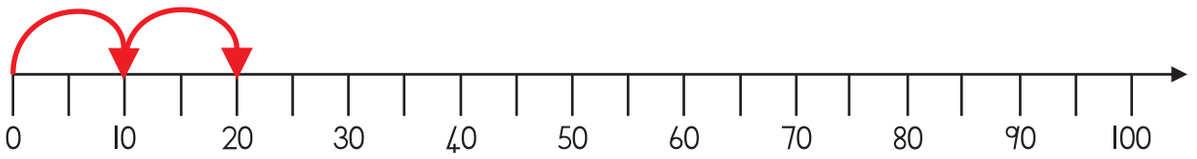
- Learners should be exposed to worksheets which present a variety of ways to do doubling and halving **(see printables)**.
- Learners will consolidate their ability to double and halve numbers by working through such activities.

Doubling and halving using a number line

- If learners need to double or halve numbers they can also use a number line to solve the problem.
- There are two worked examples below which you should work through with your learners to show how this is done.
- There is a printable at the end of this item which you can then give to your learners to complete to consolidate the use of a number line when doubling/halving **(see printables)**.

Example 1: Double 10 is _____

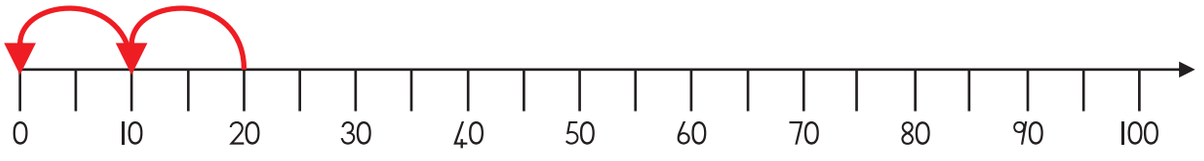
- Learners should understand how to use a number line when doubling.
- Indicate what is given: in this case, 10. Then double it, which means add the same number.
- On a number line learners can see that double means to add the same number.



Double 10 is 20
(10 + 10 = 20)

Example 2: Half of 20 is _____

- Learners should understand how to use a number line when halving.
- Indicate what is given: in this case, 20. When halving, the same number must be subtracted twice to get to zero.
- When we find the number to subtract twice to give us zero, we have found half of the original number.



Half of 20 is 10
(20 - 10 = 10) or (20 - 10 - 10 = 0)

Another example of how knowledge of doubling and halving can be tested

ANA 2014 Grade 3 Mathematics Item 3

3. 37 doubled =

A 78
B 67
C 74
D 66

Notes:

Printable: Number cards

1	2	3	4
5	6	7	8
9	10	11	12

13	14	15	16
17	18	19	20
21	22	23	24

37	38	39	40
41	42	43	44
45	46	47	48

49	50	51	52
53	54	55	56
57	58	59	60

Printable: Doubling and halving

Complete the table

Double	
135	
64	
340	

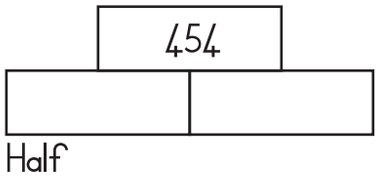
Half	
450	
118	
84	

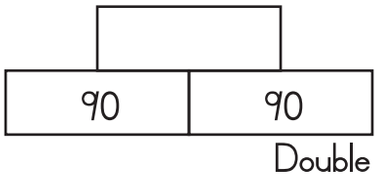
Complete the following:

Double 5	
Double 120	
Double 300	
Double 54	

Half of 10	
Half of 420	
Half of 60	
Half of 70	

Find the half or the double of a number

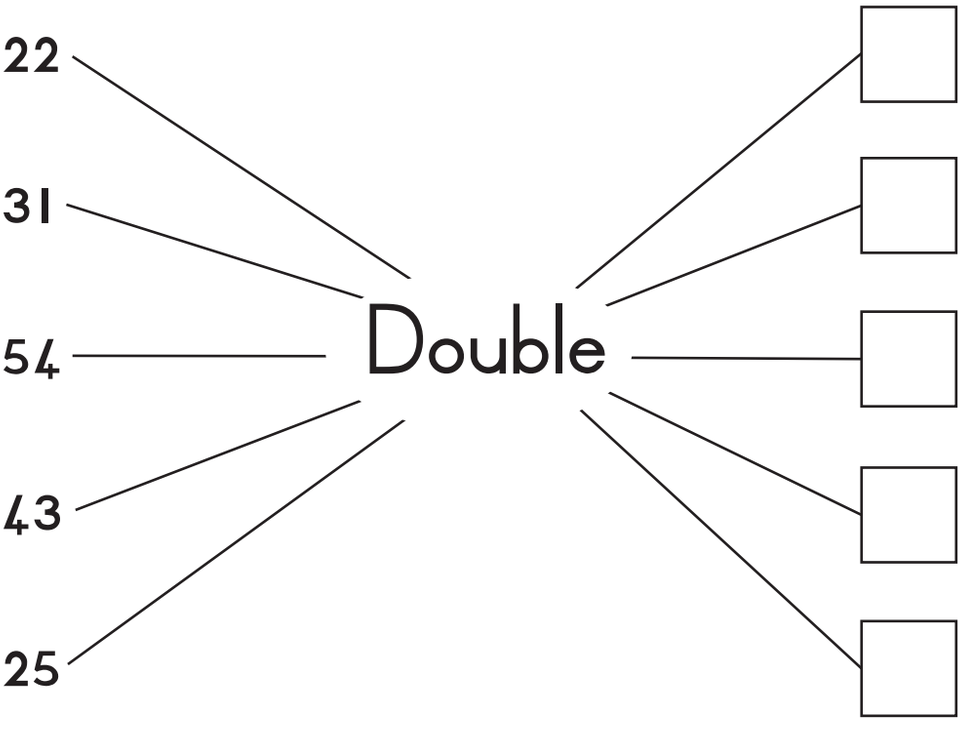


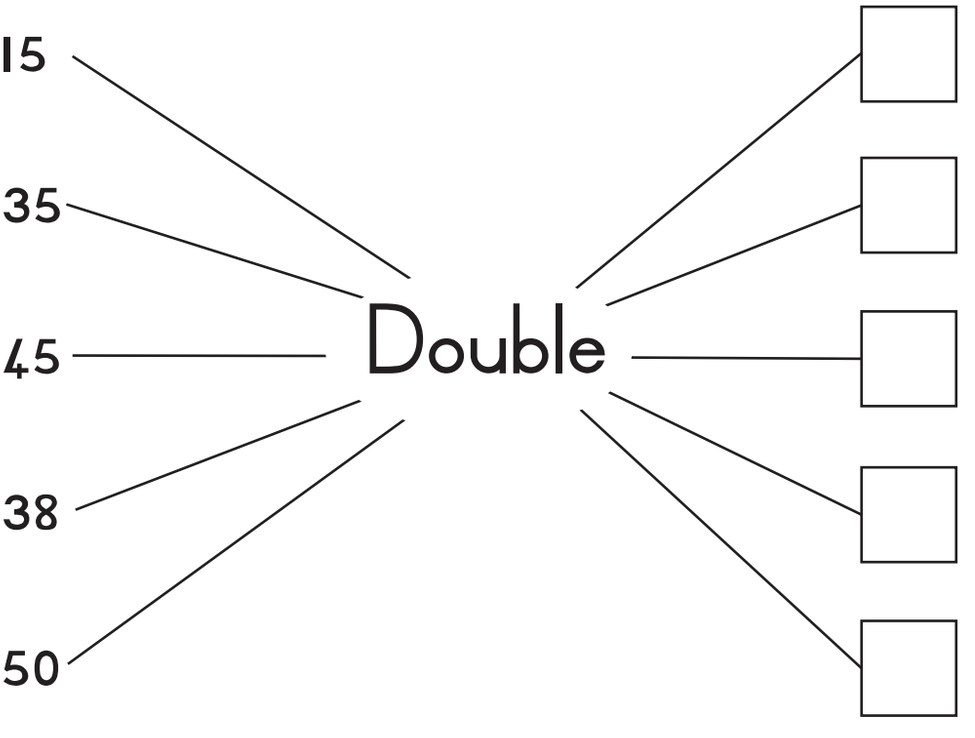


Double the numbers:

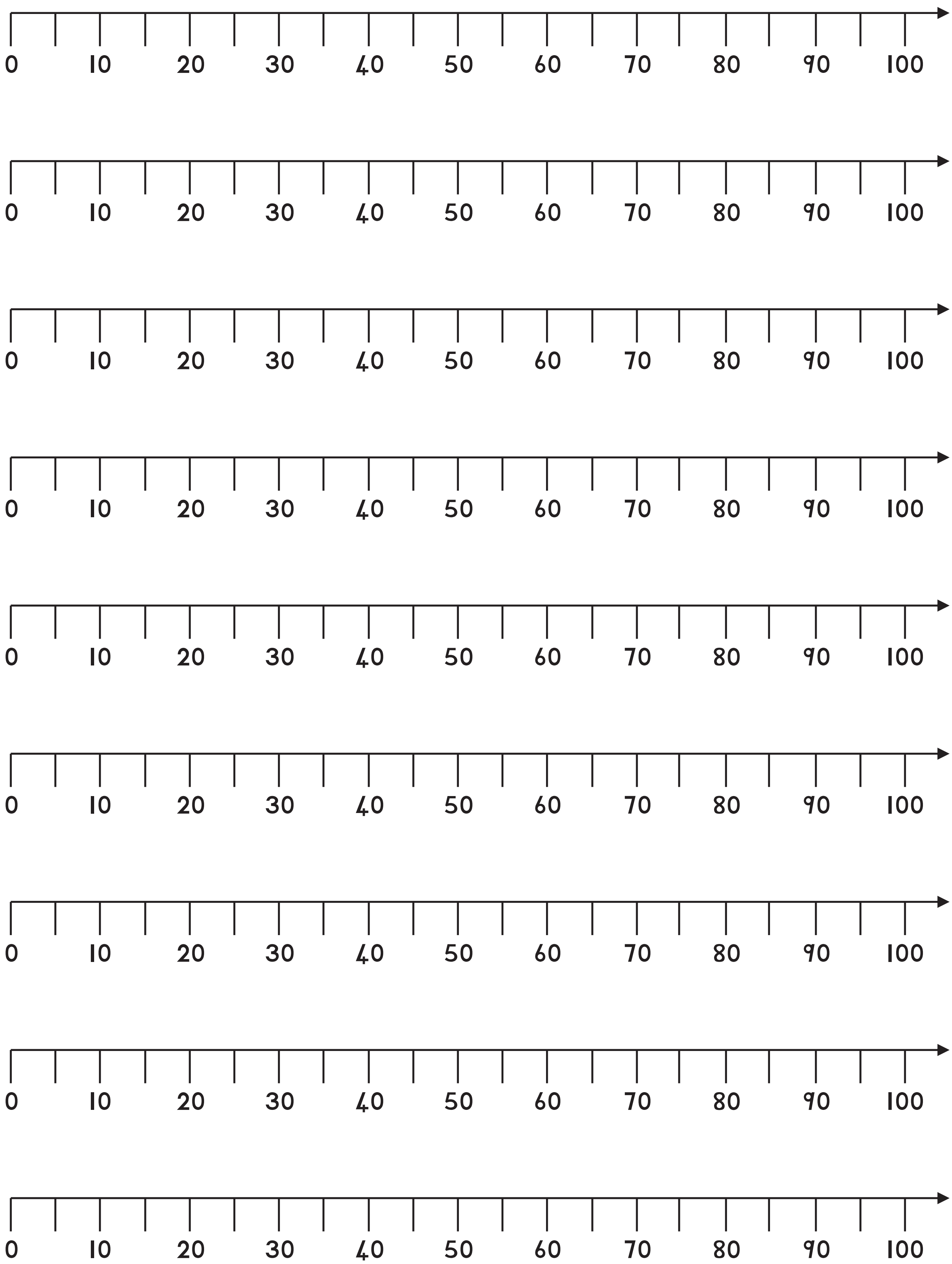
5 →	
4 →	
2 →	
3 →	

Double the numbers:





Printable: Number lines



Rounding off in tens

ANA 2013 Grade 3 Mathematics Item 4

4. Round off 642 to the nearest 10.

A 700

B 655

C 640

D 600

What should a learner know to answer this question correctly?

Learners should be able to:

- Understand place value and recognise the value of each digit in a three digit number;
- Identify the tens that are respectively smaller than and greater than the given number;
- Understand the concept of estimation;
- Understand the mathematical vocabulary: nearest.

Where is this topic located in the curriculum? Grade 3 Term 2

Content area: Numbers, Operations and Relationships.

Topic: Solve problems in context: Problem-solving techniques.

Concepts and skills:

- Rounding off in tens.

What would show evidence of full understanding?

- If the learner gave the answer C (640): This shows the learner has an understanding of place value and recognises the ten in a three digit number and can identify which number is the closest ten to the given number.

What would show evidence of partial understanding?

- If the learner selected answer D: This shows the learner did not correctly identify the ten in the given three digit number and so could not round off correctly to the nearest ten.
- However, the learner did recognise that 600 is the nearest hundred, as opposed to 700 (answer A). This shows an understanding of the process of rounding off.

What would show evidence of no understanding?

- If the learner gave the answer A (700) or B (655):
  - In answer A, the learner rounded off to the larger hundred, thereby showing an inability to recognise that the nearest ten to the given number was required as the answer.
  - In answer B, the learner was unable to round off to the nearest ten as he/she selected an answer in which there were five units represented.
  - If the learner selected either of these two answers, it shows a lack of understanding of place value (an inability to identify the ten), as well as a lack of understanding of the word "nearest".

What do the item statistics tell us?

60% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- The distractor provided by answer D may have made the item more difficult as learners may have become unsure as to whether they needed to identify the nearest ten or the nearest hundred.

Teaching strategies

Rounding off in tens means to change a number to the nearest ten. If a given number ends in 5 or more, the number is rounded up to the ten that is above the given number. If the given number ends in a number less than 5, the number is rounded down to the ten below the given number. For example, 67 will be rounded up to 70, but 63 will be rounded down to 60; 582 will be rounded off to 580 but 588 will be rounded off to 590.

Rounding off walk

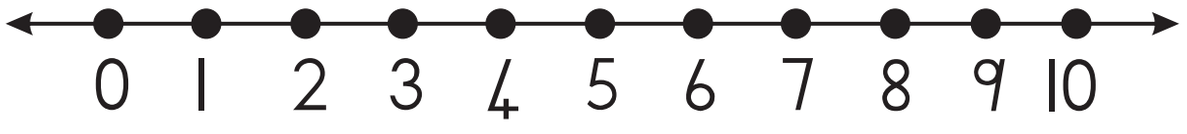
- Work with one group of learners at a time whilst the remainder of the class works independently on set classwork. Allow time for each group in the class to have a turn working with you on this activity.
- Find a relatively large space to work in, either in the classroom or the corridor outside the classroom.
- Use chalk to draw a large set of eleven squares on the floor. These squares need to be big enough for the children to stand in.

- Number the squares 0 – 10.
- Place a picture of (or draw) a coloured house at each end of the row of squares. Use a different colour for each house. See the example that follows:

- Explain to the learners that the squares are like a pathway that they will walk on, with a friend's house at either end of the path.
- Ask a learner to stand on the number 7.
- Tell the learner that he/she has taken a walk on the pathway, but it has suddenly started to rain. Tell the learner that he/she needs to get out of the rain as fast as possible. Explain to the learners "We are going to see if it is quicker to go to the blue house or the red house".
- Ask the learner to walk to the number 0 square, counting his/her steps aloud as he/she walks. The learner will walk and count "1, 2, 3, 4, 5, 6, 7".
- Ask the learners "How many steps did it take (learner's name) to get to number 0?" The learners will respond "It took 7 steps to get to number 0".
- Ask the learner to go back to number 7 and to then walk to number 10, counting his/her steps aloud as he/she walks. The learner will walk and count "1, 2, 3".
- Ask the learners "How many steps did it take (learner's name) to get to number 10?" The learners will respond "It took 3 steps to get to number 10".
- Ask the learners "Was it shorter to walk to the 0 or to the 10?" The learners will respond that it was shorter to walk to the 10.
- Explain to the learners that this means it would be quicker to walk from number 7 to the red house to get out of the rain.
- Repeat the above steps with a variety of numbers.
- Lead learners to the understanding that when standing on any of the numbers from 5 to 10 it would be quicker to go towards the red house, but when standing on the numbers from 0 to 4 it would be quicker to go to the blue house.

Rounding off using a 0 – 10 number line

- Draw a 0 – 10 number line on the board, as shown below.



- Ask learners to draw the same number line on their white boards or scrap paper.
- Ask learners to point to the number 3 on their number lines.
- Ask the learners "Is the number 3 closer to the arrow on the zero side of the line or to the arrow on

the ten side of the line?" Learners will say that the 3 is closer to the zero side.

- Ask them to count how many jumps they need to get to the zero. Learners count "1, 2, 3: three jumps".
- Ask learners to count how many jumps they need to make to get to the ten side of the line. Learners count "1, 2, 3, 4, 5, 6, 7: seven jumps".
- Repeat the above steps with other numbers.

Rounding off using a 10 – 20 number line

- Draw a 10 – 20 number line on the board, as shown below.

- Ask the learners to put a finger on the number 15.
- Ask learners "From the number 16, is it closer to the 10 or to the 20?" Ask learners to explain how they got their answer. Learners may respond by saying "16 is closer to 20 because I only had to jump 4 places to 20 but 6 places to 10".
- Ask learners "So which ten is closer to 16 – ten or twenty?" Learners will say "20".
- Repeat the above steps with a variety of numbers.
- Lead learners to the understanding that we say that the numbers 15 to 19 are closer to 20, but that the numbers 11 to 14 are closer to 10.
- Explain to learners that "Sometimes it is easier to only talk about the tens and to forget about the ones for a moment. So we say that we round off numbers to the closest ten. This means that the numbers 15 to 19 would be rounded off to 20 because they are closer to 20 than they are to 10 and the numbers 11 to 14 would be rounded off to 10 because they are closer to 10 than they are to 20. We can do this for all the different tens."

Rounding off using tens cards

- Ask learners to work in pairs or groups of four.
- Hand out a set of cut up tens cards to each pair or group of four **(see printables).**
- Write the number 67 on the board.
- Ask learners to find the ten before the number 67 and the ten after the number 67 and to hold up those two cards. Learners will hold up the 60 and 70 cards. Allow learners to refer to their 1 – 100 number boards if necessary
- Ask learners "How did you know you needed to hold up those two cards?" Learners will respond "On our number boards we can see that the ten that comes before 67 is 60 and the ten that comes after 67 is 70".
- Tell learners to "Hold up the ten that is closest to the number 67". Learners should hold up the 70

card.

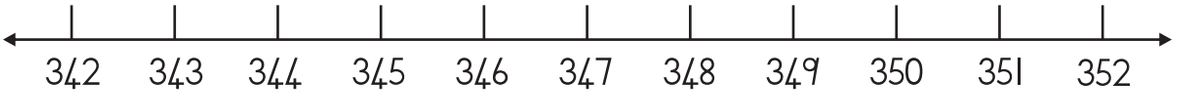
- Ask learners "How did you know that 67 is closer to 70 than to 60?" Learners may say "On our number boards we have to jump 3 places to 70, but 7 places to 60, so 70 is closer". Some learners may have remembered the previous lessons and so respond "Numbers 65, 66, 67, 68 and 69 would be rounded off to 70 and numbers 61, 62, 63 and 64 would be rounded off to 60".

Rounding off in the number range to 999 using number lines

Group work activity:

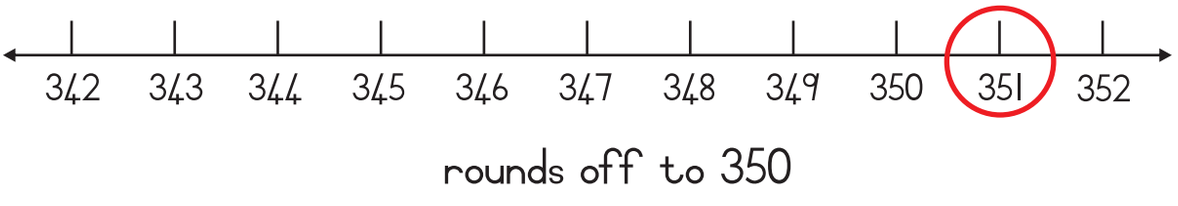
- Ask learners to work in groups of four.
- Give each learner a sheet of blank number lines **(see printables)**.
- Each learner must select a number range for his/her number line and fill in the missing numbers.
- Encourage learners to select number ranges that they are comfortable with, including those in the hundreds.

  For example:



- Each learner must then select a number from his/her number line to round off.
- Each learner must then tell the rest of the group which number he/she selected and to which ten it would be rounded off.

  For example:



- Learners can then swop their number lines within their groups of four and repeat the above steps choosing different numbers.

Individual activity

- Learners in Grade 3 can be given a rounding off worksheet **(see printables)** to consolidate their understanding of rounding off in higher number ranges.
- The worksheet has a variety of number ranges so that learners can fully develop their ability to round off numbers.
- Once learners have completed the worksheet allow them time to share their answers in their groups or in the class. They should notice that numbers can round up or down to a given number. For example, 351 rounds off to 350 and so does 348.

Another example of how rounding off can be tested

ANA 2014 Grade 3 Mathematics Item 5

5. Round off 132 to the nearest 10.

A 140

B 135

C 130

D 100

Notes:

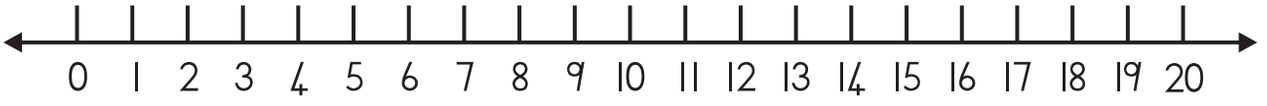
Printable: Tens cards

10	60
20	70
30	80
40	90
50	

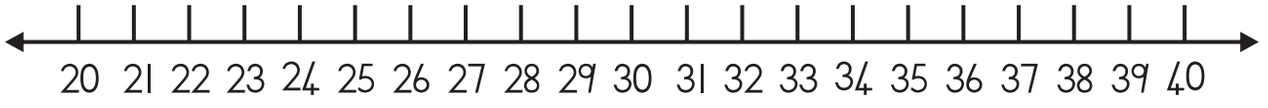
Printable: Number lines

Printable: Rounding off

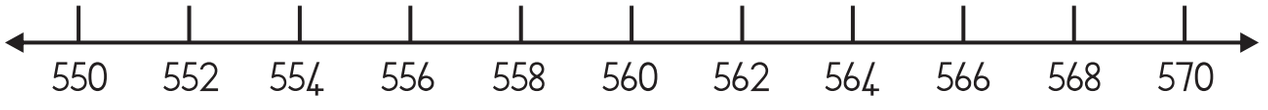
Circle three numbers that will round off to 20 on the number line below



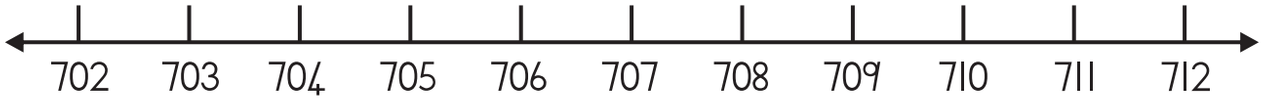
Circle three numbers that will round off to 30 on the number line below



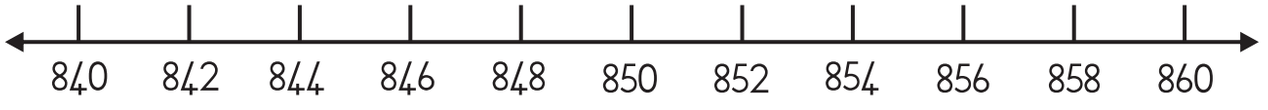
Circle three numbers that will round off to 560 on the number line below



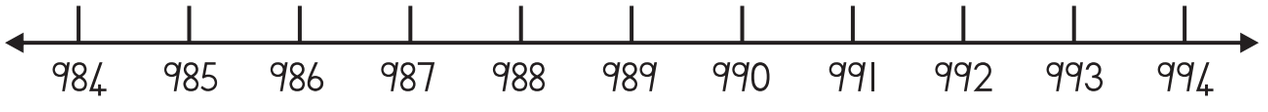
Circle three numbers that will round off to 710 on the number line below



Circle three numbers that will round off to 850 on the number line below



Circle three numbers that will round off to 990 on the number line below



Fractions

ANA 2013 Grade 3 Mathematics Items 6 and 21

6. Arrange 1 half, 1 quarter, 1 third, 1 fifth from the greatest to the smallest.

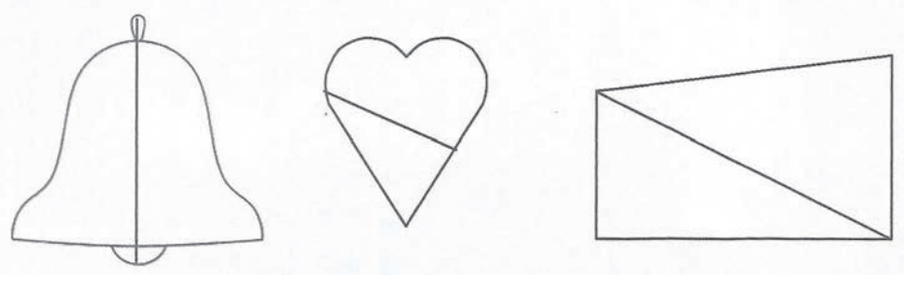
   A. 1 quarter, 1 half, 1 fifth, 1 third

   B. 1 half, 1 third, 1 quarter, 1 fifth

   C. 1 third, 1 fifth, 1 quarter, 1 half

   D. 1 fifth, 1 quarter, 1 third, 1 half

21. Circle the shape that is divided in half.



What should a learner know to answer these questions correctly?

Learners should be able to:

Item 6

- Read and understand the word names for fractions;
- Have a method of comparing the relative sizes of unit fractions;
- Know the meaning of the words greatest and smallest; and
- Arrange the fractions in order.

Item 21

- Recognise the terminology: divide and half;
- Understand that one half of a shape must be exactly the same as the other half;

- Link this understanding of fractions to knowledge of symmetry.

Where is this topic located in the curriculum? Grade 3 Term 1 - 4

Content area: Numbers, Operations and Relationships.

Topic: Fractions.

Concepts and skills:

- Item 6 (Term 4): Order, describe and compare fractions.
- Item 21 (Term 1): Recognise fractions in diagrammatic form.

What would show evidence of full understanding?

Item 6

- If the learner gave the answer B: this shows the learner ordered the fractions correctly and has the required knowledge and understanding.

Item 21

- If the learner selected the bell shape: this shows the learner recognises that shape must be divided into two halves that are exactly the same.

What would show evidence of partial understanding?

Item 6

- If the learner gave the answer D:
  - The learner ordered the fractions, but in the wrong way (i.e. from smallest to greatest instead of greatest to smallest as required by the question).
  - The learner demonstrated an ability to compare the relative sizes of the fractions, but did so in the wrong order.
  - The learner's knowledge of 'greatest' and 'smallest' may need further development.

Item 21

- If the learner circled the heart or the quadrilateral: this shows the learner recognised that the shape had to be divided into two parts, but did not know that the two parts had to be equal in size in order for the shape to be divided in half.

What would show evidence of no understanding?

Item 6

- If the learner gave the answer A or C: in these options the fractions have not been ordered at all. This shows the learner does not know how to order fractions from greatest to smallest.

Item 21

- If no answer was given: this could demonstrate no understanding or a lack of time or carelessness on the part of the learner.

What do the item statistics tell us?

Item 6

40% of learners answered the question correctly.

Item 21

74% of learners answered the question correctly.

Factors contributing to the difficulty of the items

Item 6

- Since this item is located in the Grade 3 Term 4 curriculum, some learners may have found it difficult. However, the fractions in the question are all unit fractions which simplifies the question to some extent. The question could have been more difficult if some non-unit fractions (e.g. 2 fifths) had been included in the list to order.

Item 21

- This item was relatively simple as it relies on knowledge covered previously in Grades 1 and 2.

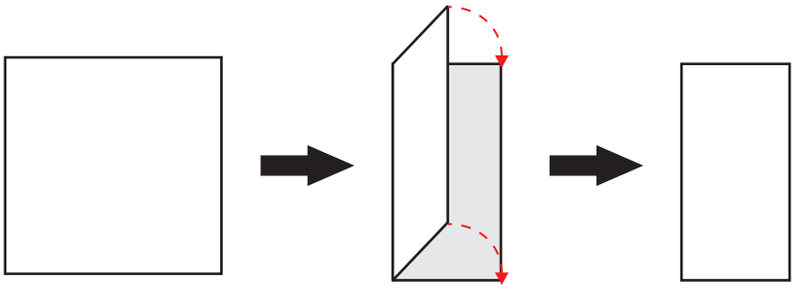
Teaching strategies

Establishing the concept of fractions – using paper folding

- Ask learners to work in pairs or groups of four.
- Give each pair/group of four a collection of paper shapes, for example a rectangle, a square, a triangle, a circle and a diamond.
- Ask learners to select the square and place it in front of them.
  - Ask learners to describe what they see in front of them. Learners may respond by saying "Paper" or "Square"
  - Model the correct language. Say "We have one shape. We have one whole shape".
  - Ask learners "How do we know it is a whole shape?" Learners should respond "There is only one piece of paper" or "The shape is not torn or cut".
  - Say "We call this a whole. Can you hold up another whole from the shapes you have on your desk?" Learners can hold up one of the other shapes.
  - Ask the learners "What do we have if I fold my square down the middle?" Learners should respond by saying "You have a small rectangle" or "You have two small rectangles".
  - Unfold the shape and ask the learners "Is it still a whole?" Learners should respond by saying "Yes. You can see the two small rectangles, but it is still one piece of paper so it is still a whole".

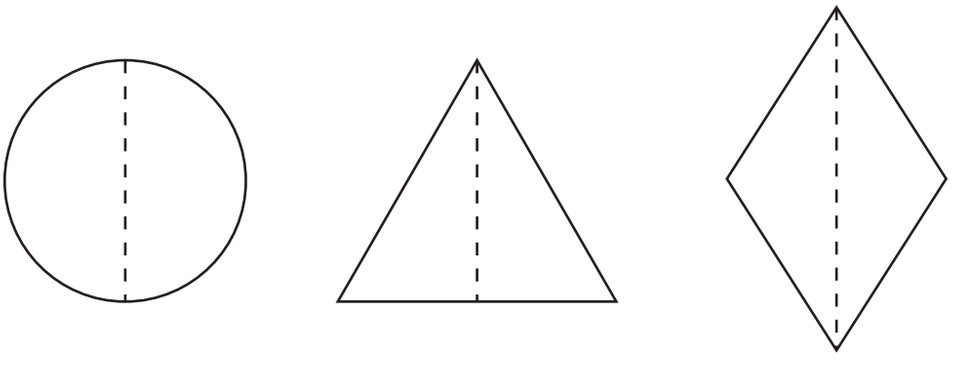
- Explain to the learners that the whole shape can be made up of two smaller shapes.

- Then ask learners to fold their squares in half, taking care to match the corners up exactly.



- Ask learners to tell you what they see. The learners should respond by saying “I see two small rectangles” or “The small rectangles are both the same size” or “A square is made up of two rectangles”.
- Say “Right, we have folded our shape in half. For it to be folded in half, the two pieces need to be exactly the same. When I have folded my shape like this I have found two fraction parts of the shape that make up the whole”.
- Then say “If I fold my shape like this (demonstrate folding over a small piece of the square), would that be folding it in half? Why do you say so?” Learners should respond by saying “No, the two pieces need to be exactly the same and one piece is much smaller than the other one”.
- Emphasise that fraction parts must be equal in size.

- Ask learners to now try folding their rectangles in half on their own.
  - Ask learners, “How do you know you've folded the shape in half?” Learners should respond by saying “The two pieces are exactly the same”.
  - Ask learners “What do you notice about the two pieces?' Encourage learners to realise that the two pieces are a different size/shape from the whole and that together the two pieces make up the whole.



- Repeat the above steps with the rest of the paper shapes.
- Talk about what you have found when you folded the paper shapes in this activity. You found fraction parts. Fractions are parts of a whole.
- For example: Half of a square is one of two equal sized parts into which a square has been folded. In

terms of numbers, a fraction is a part of a number. Half of 1 is $\frac{1}{2}$.

- Working with a fraction board
- Place learners in pairs or groups of four and give each pair or group a fraction board cut into strips **(see printables)**.
  - Ask learners to hold up the strip that they think represents the whole. Ask them "Why did you choose that strip?" Learners should respond by saying "This strip shows 1 shape/piece. You cannot see the different parts that make it up".
  - Then ask learners if they can hold up the strip that shows the shape divided in half. Ask them "How do you know this strip shows half?" Learners should respond by saying "When you divide something in half it has two pieces that are exactly the same".
  - Ask learners to place the halves strip directly on top of the whole strip and ask them "What do you notice?" Learners should say "Two halves are exactly the same size as one whole" or "One whole is made up of two halves".
  - Ask learners to fold the halves strip on the halving line and to then place the folded paper (one half) on top of the whole. Ask them "What do you notice?" Learners should respond by saying "One half covers up half of the whole" or "The uncovered side is exactly the same size as the covered side" or "We have covered one half of the whole".

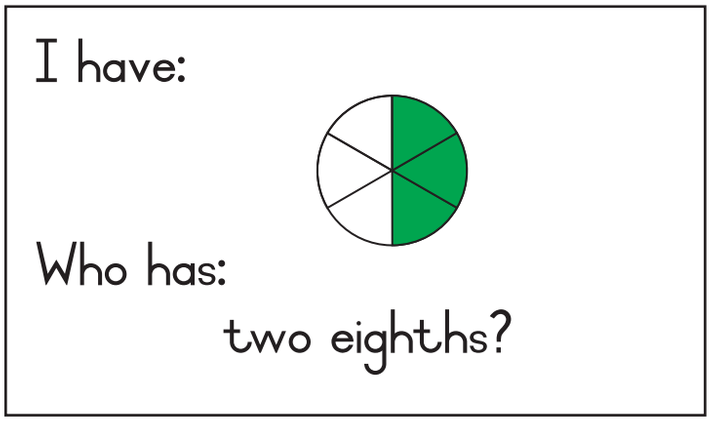
- Repeat the above steps with thirds, quarters, fifths, sixths and eighths by folding them on the lines and placing them onto the whole.
- Then repeat the above steps but place the quarters strip on top of the halves strip so that learners can discover that one half is equivalent to two quarters.

Working with parts of the whole

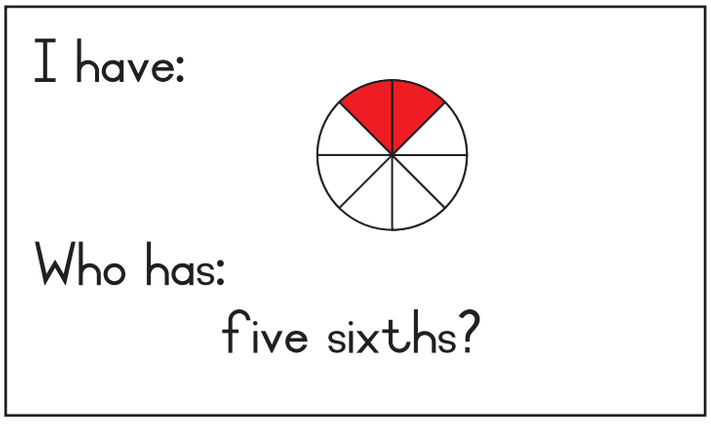
- Ask learners to work in groups of five or ten.
- Shuffle the cut-up 'I have … Who has' cards well **(see printables)**.
- Hand out the cards. Learners will each get either four cards (for the group of five) or two cards (for the group of ten).
- Learners need to be able to recognise the diagrammatic representation of the fraction in order to do this activity. For this reason, you may need to facilitate the activity in the beginning so as to help the learners interpret the diagrams. With practice and greater knowledge of fractions, this could be a game played by groups of learners independently of the teacher

.

- One learner starts by reading out the card, e.g. “I have three sixths. Who has two eighths?”



- The other learners in the group must then look through the cards in their hand and see if they have the diagram answering the question “Who has two eighths?”



- The person who has the correct card (as shown below) then reads out correct the card: “I have two eighths. Who has five sixths?”
- This process continues until all the cards have been read.
- If time allows, the cards could then be re-shuffled and handed out so as to play the game again.

Another example of how fractions can be tested

ANA 2014 Grade 3 Mathematics Item 17

17. Answer questions 17.1 and 17.2.

17.1 Nelson eats 2 pieces of the chocolate shown below.

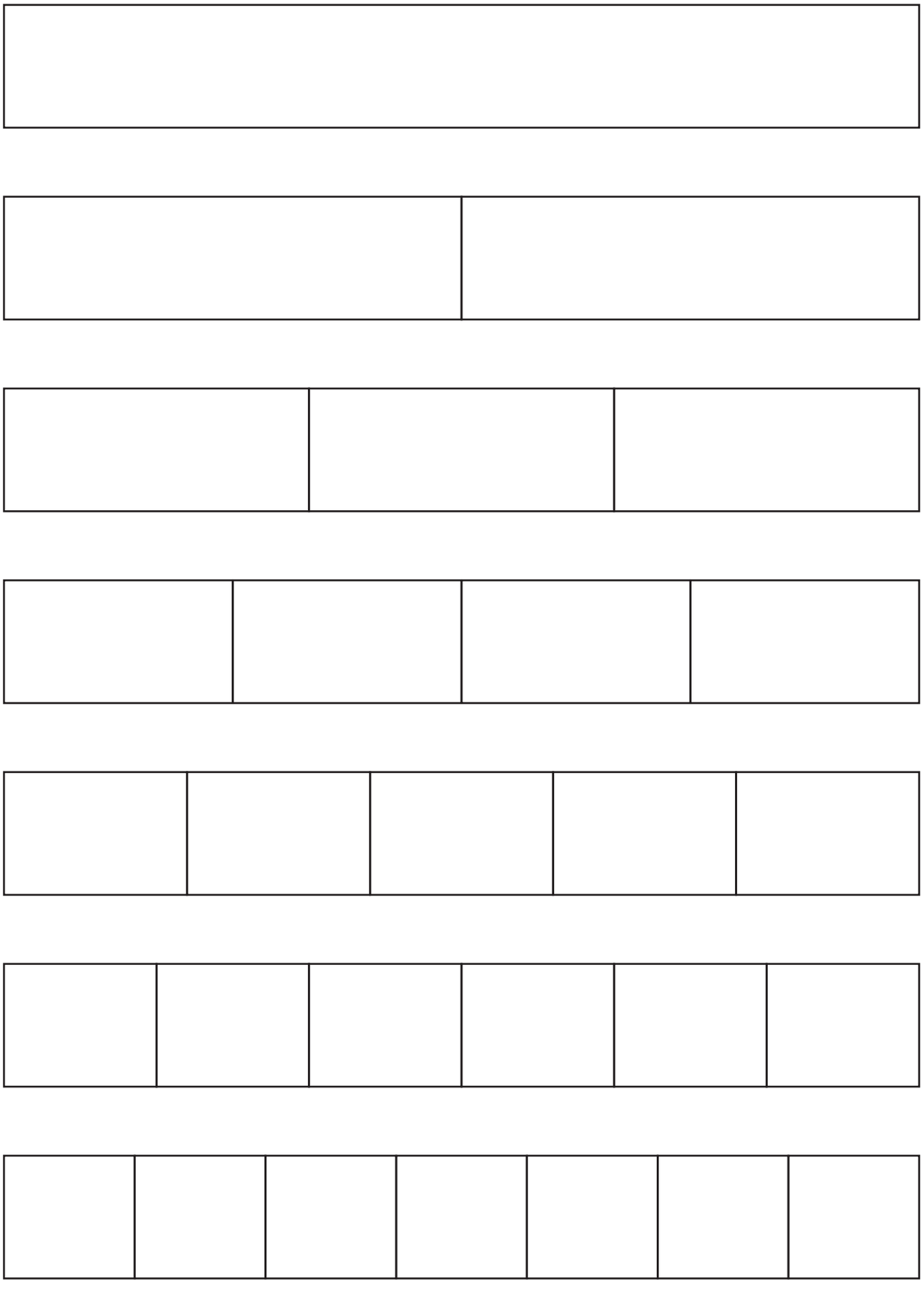
What fraction of the chocolate did Nelson eat? _____

17.2 Zinzi eats a quarter of the chocolate shown below.

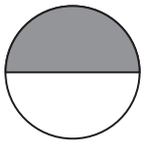
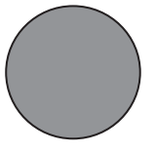
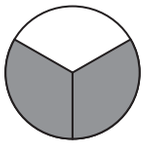
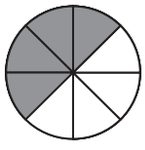
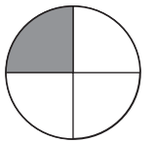
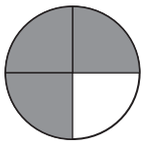
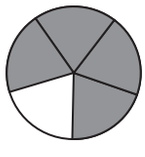
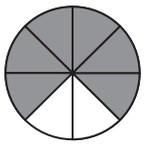
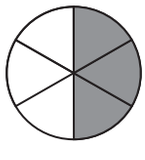
How many pieces did Zinzi eat? _____

Notes:

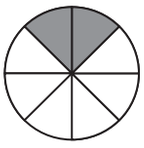
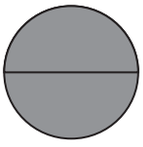
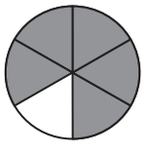
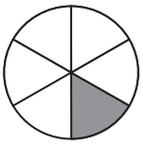
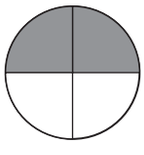
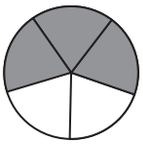
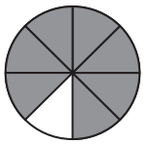
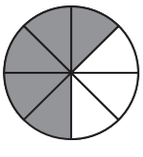
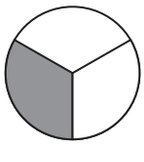
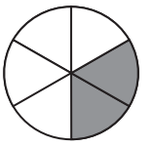
Printable: Fraction board



Printable: "I have … Who has" cards (1)

I have:  Who has: two thirds?	I have:  Who has: four eighths?
I have:  Who has: one fifth?	I have:  Who has: one quarter?
I have:  Who has: three quarters?	I have:  Who has: four fifths?
I have:  Who has: six eighths?	I have:  Who has: three sixths?
I have:  Who has: one whole?	I have:  Who has: two eighths?

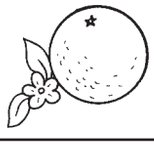
Printable: "I have … Who has" cards (2)

I have:  Who has: five sixths?	I have:  Who has: one sixth?
I have:  Who has: two quarters?	I have:  Who has: three fifths?
I have:  Who has: seven eighths?	I have:  Who has: five eighths?
I have:  Who has: one third?	I have:  Who has: two sixths?
I have:  Who has: two halves?	I have:  Who has: one half?

Money: addition and subtraction

ANA 2013 Grade 3 Mathematics Items 24.1 and 24.2

24. Read the price list below and answer the questions that follow.

Price list		
Bunch of grapes	R5,50	
Pineapple	R10,00	
Apple	R5,50	
Orange	R6,00	

24.1 How much will 2 pineapples cost?

R _____

24.2 How much change must I get if I buy one orange and pay with R10.00?

R _____

What should a learner know to answer this question correctly?

Learners should be able to:

- Interpret the questions asked in order to identify which operation is needed to solve the problem;
- Disregard irrelevant information so as to simplify the item;
- Understand the concept of change as being that which is given when too much money is paid initially;
- Add or subtract rand amounts by using the correct calculation strategies;
- Show the method they used to solve the problems.

Where is this topic located in the curriculum? Grade 2 Term 1

Content area: Numbers, Operations and Relationships.

Topic: Money.

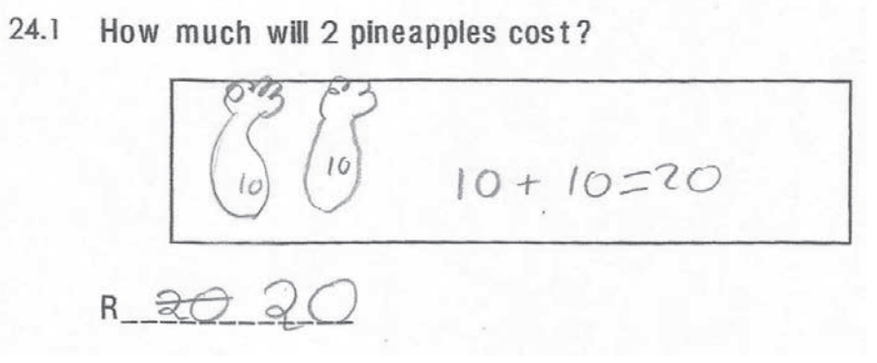
Concepts and skills:

- Solve money problems involving totals and change in cents up to 50c and rands up to R20.

What would show evidence of full understanding?

Item 24.1

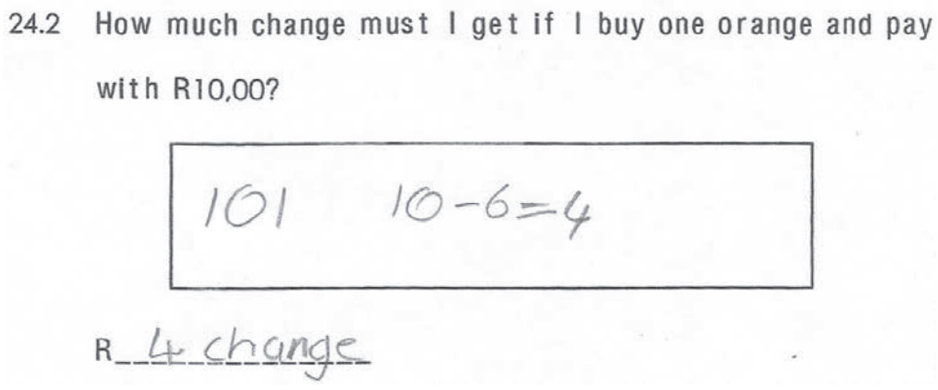
- If the learner answered R20 and showed the correct working as shown in the response below;
  - The learner decided to draw the two pineapples as part of the solution strategy. The prices of the pineapples were written on the fruits, showing an awareness that each pineapple cost R10.00;
  - The learner was then able to write out the addition algorithm 10 + 10 = 20 and provide the answer R20.



Item 24.2

- If the learner answered R4 and showed the correct working as shown in the following response:
  - The learner was able to read the problem and determine which operation to use;
  - The numbers written on the left hand side of the block (101) suggest that the learner attempted to highlight the key information needed to solve the problem. The learner may have written 10 to show the amount of money that was used to pay. The learner may then have written a 1 next to the 10 to show that only 1 orange was bought;

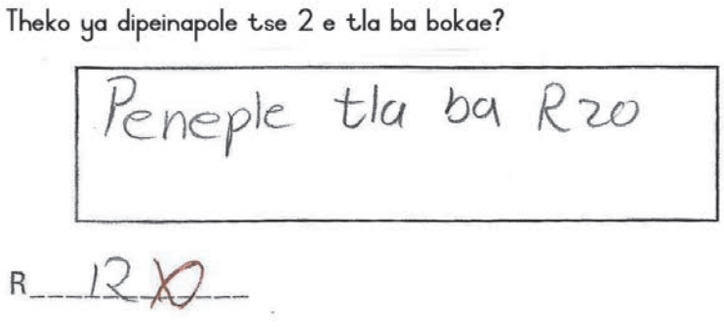
- The learner was then able to write the correct algorithm, 10 – 6 = 4 and calculate that R4 change needed to be given.



What would show evidence of partial understanding?

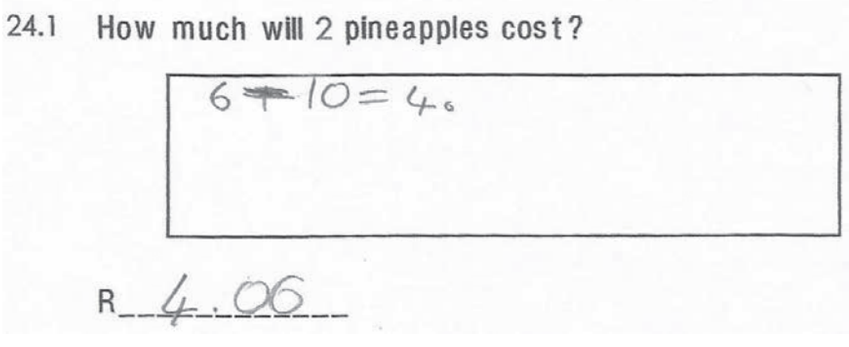
Item 24.1

- If the learner provided the response below: this shows the learner understood correctly that two pineapples cost R20. However, the learner was unable to write the correct answer and wrote R12 instead of R20.



Item 24.2

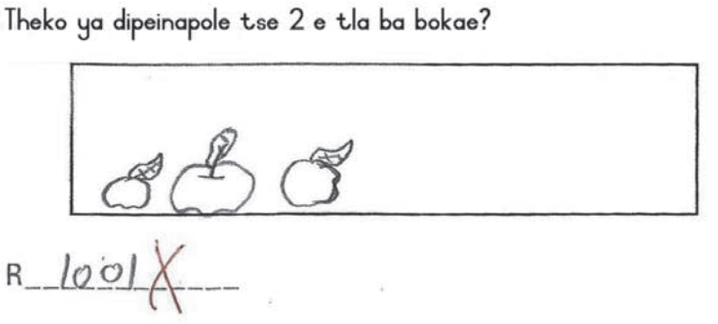
- If the learner gave the correct answer, but did not show how he/she got to that answer;
- In the following example the learner wrote the algorithm as 6 – 10 = 4, which shows some confusion in terms of how to write out a number sentence. However, the learner gave the correct answer, R4.



What would show evidence of no understanding?

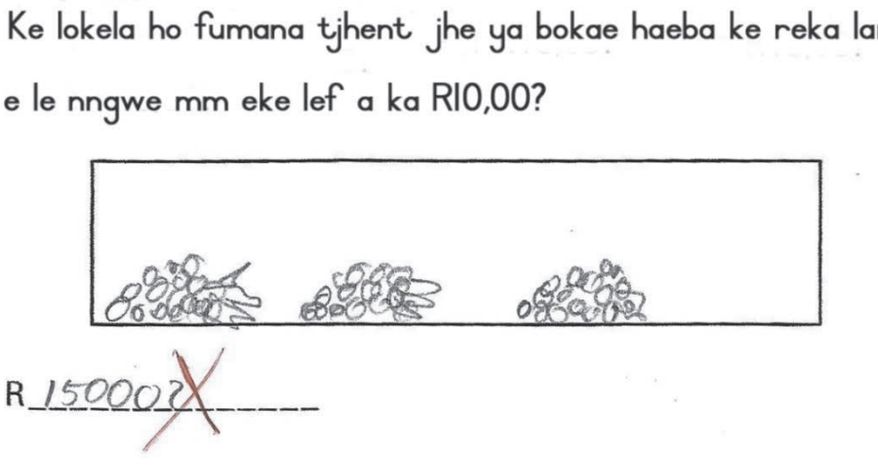
Item 24.1

- The response below shows no understanding;
- It appears that the learner did not understand the language in the problem as the learner drew three apples instead of two pineapples;
- The learner was unable to recognise that the price of each of the two pineapples needed to be added together in order to determine the total cost;
- The learner wrote the answer as R100c which shows that the learner did not understand what to do with the given information.



Item 24.2

- The response below shows no understanding;
  - It appears that the learner did not understand the language in the problem as the learner drew three bunches of grapes instead of one orange;
  - The learner was not able to determine that this is a subtraction problem and that R6,00 needed to be taken away from R10,00 in order to calculate the change needed.
  - The answer was given as R15000c which indicates that the learner has no understanding of the value of money.



What do the item statistics tell us?

Item 24.1

56% of learners answered the question correctly.

Item 24.2

47% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Learners may have found this item more difficult as the rand amounts were written with a comma and zeros as place holders for the cents (for example: R10.00). In many instances in the classroom, learners would have seen rand amounts written more simply, without the cents (for example: R10).
- The amount of irrelevant information may also have made this problem more difficult for learners as they could have become unsure of what information they needed to focus on.
- The level of reading required in this problem could also have caused difficulties, resulting in learners being unsure of which fruit to select and then not knowing what needed to be done in order to solve the problem.

Teaching strategies

Paper money calculations

- Ask learners to work in pairs or groups of four.
- Provide each pair or group of four with cut up paper money **(see printables).**
- Begin this activity by focusing on coin recognition to ensure that the learners know what each coin looks like and what amount it represents.

- Ask learners to hold up a 10c coin.
  - Ask them, “How do you know this is a 10c coin?” Learners can talk about the colour, size, picture, numbers and the c (for cents) that they see on the coin.
- Repeat with all the coins, taking care to compare the coins and to talk about their similarities and differences.

- Ask learners to hold up the R5 coin and the 5c coin.
  - Ask the learners “Which is the R5 coin?” Learners should respond by describing the features of the coin.
  - Ask learners “How did you know that the other coin isn't the R5 coin?” Learners should respond that on the R5 coin they can see the word 'rand' and on the 5c coin they can see the 'c' which stands for cents.
  - Ask learners “Which coin is worth more?” Learners should respond that the R5 coin is worth more.

- Ask learners if they can show you R2 without using a R2 coin.
  - Learners will hold up a two R1 coins or one R1 and two 50c coins, etc.

- Ask learners to add R3 to the R2 that they put aside. Ask them "Do we have a R3 coin that we can use?"
  - Learners should respond "No" and will then show the coins that they used instead (one R2 coin and one R1 coin or three R1 coins, etc.).

- Ask learners "How much money do you have altogether now?" Learners should respond that they have R5.
  - Ask learners to take away R1 from the R5 that they put aside;
  - Ask the learners "If you take R1 away from R5, how much money do you have left over?" Learners should respond that they have R4 left over.

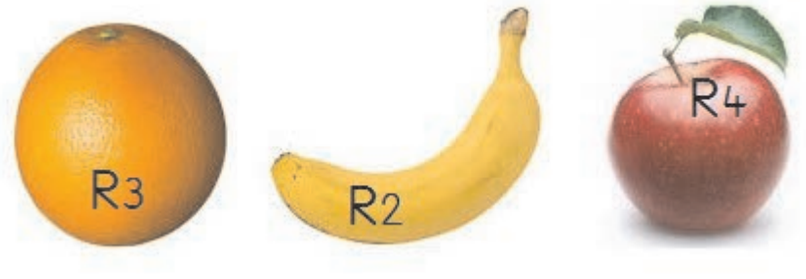
- Repeat with many different amounts of money and combinations of coins.
- Learners need much practice with adding and subtracting money and it is necessary for them to develop their understanding of this through working concretely with paper cut-outs of money.
- In making up amounts of money use multiple rather than single coins so that learners are able to physically take away coins and count the remaining money. This is more meaningful for them than trying to subtract (for example) R2 from a R5 coin, which may seem impossible because the R5 coin cannot be broken up into parts.

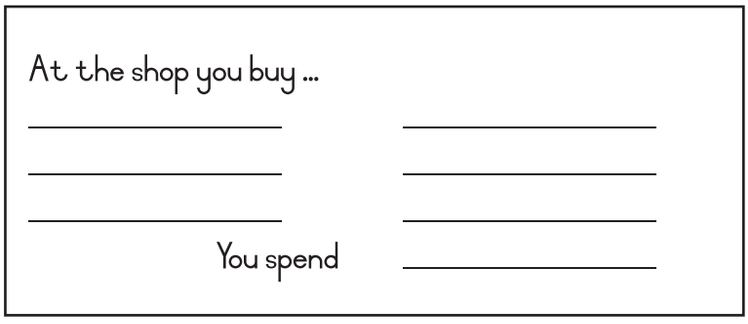
Shopping

- In preparation for this activity, cut out items from grocery store advertisements.
- Ensure that you cut the store price off the items so that you can write your own prices using numbers from a suitable number range.
- It is more important for the prices to be in a number range suitable for your learners than for the prices to be realistic.
- Write prices onto your cut out items in rands or cents. Only use combinations of rands and cents when your learners have had much practice adding and subtracting rands and cents separately.

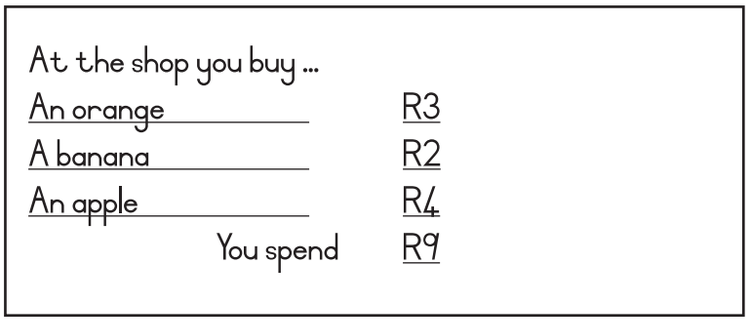
For example:

- Ask learners to work in pairs or groups of four.
- Provide each pair or group of four with a shopping list and a variety of priced items as shown below.





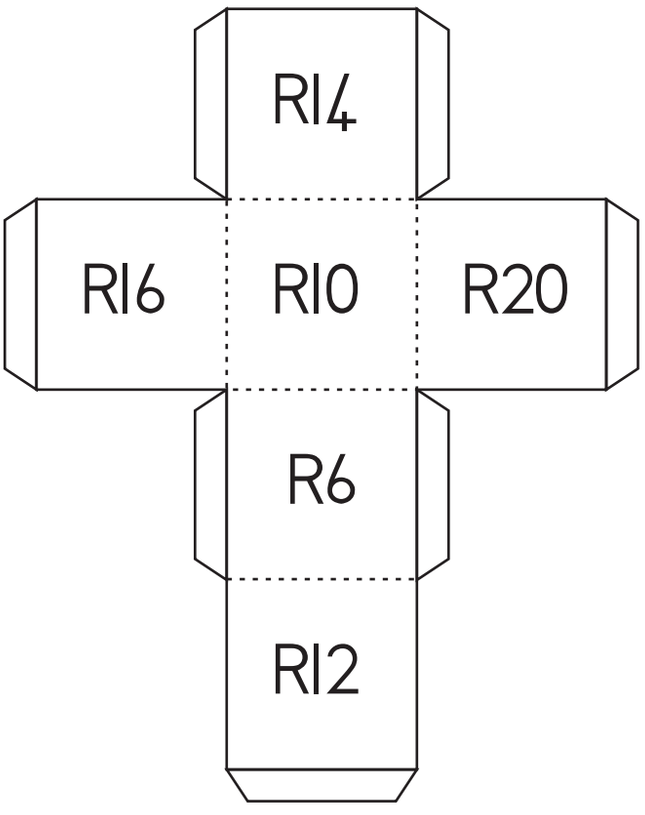
- Ask learners to select a number of priced items. The quantity depends on the ability level of the learners.
- Ask learners to complete the shopping list by writing in the name of the item and its price. The written work can be completed on scrap paper, whiteboards, or in the learners' books.



- Ask the learners to calculate how much money they would spend if they bought each of the selected priced items and to write the total in the space provided.
- Ask the learners a variety of questions, such as:
  - "How many items did you buy?"
  - "How much did they each cost?"
  - "How much did you have to spend in total?"
  - "How did you work out how much you needed to spend?"
- Allow the learners to do numerous examples.

Change dice

- For this activity you will need to use a variety of priced items as described in the activity above. The priced items for this activity need to have a monetary value of R5 or less.
- You will also need a money dice with rand amounts of R6 or larger on the faces of the dice, as shown below.



- Ask learners to work in pairs or groups of four.
- Provide each pair or group of four with a selection of priced items, a money dice and scrap paper or a white board.
- Ask the learners to select one priced item, for example: an apple

- Ask learners to roll the money dice and to take note of the amount of money shown on the face that lands on top, for example:

- Explain to learners that this means that they buy an apple for R4 and they pay for it using R10. Ask them "How much change will you need to get?"
- Ask the learners to work out how much change they will get on their scrap paper or white boards.
- Ask the learners "How did you work out how much change you needed?" Encourage learners to verbalise that R4 needs to be taken away from R10 to find the amount of change needed.
- Encourage learners to write the algorithm R10 – R4 = R6.
- Repeat this activity multiple times in order for learners to develop their understanding of the concept of change.

Notes:

Another example of how addition and subtraction in the context of money can be tested

ANA 2014 Grade 3 Mathematics Item 24

24. Read the price list below and answer the questions that follow.

		
R25.00	RI3.50	RI8.25
Drum	Guitar	Trumpet

24.1 Which two musical instruments can you buy for exactly R38.50?

_____

24.2 Jack buys a trumpet and pays with a R50 note.
How much change will he get?

He will get R _____ . _____

Printable: Coin cut-outs

Word problem: multiplication of a two digit number by a 1-digit number

ANA 2013 Grade 3 Mathematics Item 19

19. John, Jim, and Joan each have 37 lollipops.
How many lollipops do they have altogether?

What should a learner know to answer this question correctly?

Learners should be able to:

- Read the question and identify the basic operation to be applied in solving the problem;
- Analyse a problem to determine what information is known and what information is still needed to solve the problem;
- Select calculation strategies;
- Understand breaking down of numbers, place value, repeated addition and multiplication;
- Understand basic mathematical vocabulary such as: each, altogether.

Where is this topic located in the curriculum? Grade 3 Term 3 and 4

Content Area: Numbers, Operations and Relationships

Topic: Repeated addition leading to multiplication

Concepts and skills:

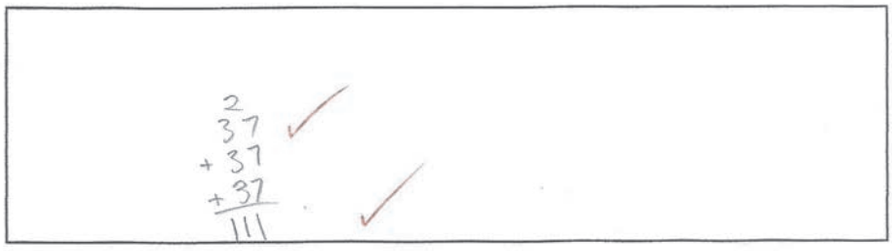
- Solve word problems in context and explain own solution to problems using multiplication with answers up to 100.

What would show evidence of full understanding?

- If the learner gave the answer 111 (different learners may have used different strategies to arrive at this answer);
- The vertical method of adding has been used, while horizontal working is still encouraged in Grade 3 in CAPS.
- This indicates the learner is able to use repeated addition of three (3) numbers, or is able to apply multiplication correctly: 3 x 37 = 111;
- It also shows the learner is able to work with numbers in tens and units to solve the problem and is able to solve the problem by carrying units (converting a group of 10 units) into tens.

19. John, Jim and Joan each have 37 lollipops.

How many lollipops do they have altogether?

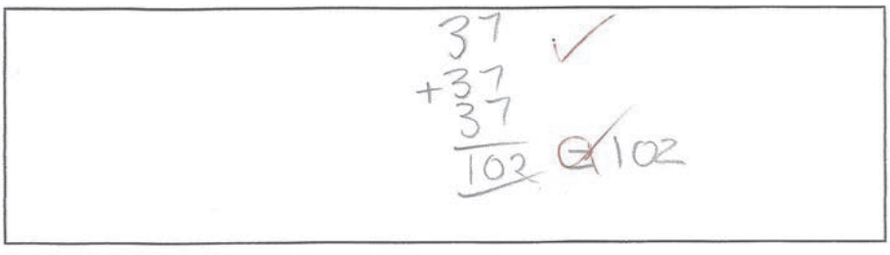


What would show evidence of partial understanding?

- If the learner realised that repeated addition or multiplication was necessary to solve the problem, but did not do the working correctly.
- The error in the working might be related to the incorrect use of place value (when you add 37 three times you need to correctly carry the units and tens).
- The learner may not know the correct algorithm to do the calculation, as shown in the example below: the learner has added three 7s to get 21, but has not moved on from this step correctly. It seems the learner wrote the 2 (tens) in the units place and carried the 1 (unit) to the tens place.

19. John, Jim and Joan each have 37 lollipops.

How many lollipops do they have altogether?



What would show evidence of no understanding?

- If the learner used the incorrect basic operation to solve the problem and gave the wrong answer; or
- If the learner gave the wrong answer and showed no method to solve the problem.

What do the item statistics tell us?

26% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- The problem was difficult for learners that struggled to read and understand the question: these learners may not have been able to determine the basic operation needed to solve the problem.
- A further factor contributing to the difficulty of the problem is that the learners were not able to add three two digit numbers together to a total of more than a hundred (100).
- Although the CAPS requires that in Term 3 in Grade 3 learners should be able to add and subtract up to 800, it also specifies that repeated addition problems in context should go up to a solution of 75 in Term 3 and up to 100 in Term 4, which may have caused some difficulty for the learners.

Teaching strategies

Solving of word problems

- You should emphasise the underlying skills required to solve word problems by giving learners multiple problem sums to solve.
- When learners are asked to solve a problem by making sense of a situation presented, they develop an awareness of what it is to add, to subtract, to multiply and to divide, rather than being told the meaning of these operations by the teacher: i.e. the learners develop conceptual understanding.
- Learners can solve problems without knowing the words addition, subtraction, multiplication and division or, for that matter, before they know the symbols representing these operations.
- Problems may be based not only on one operation but may make use of a combination of operations.
- When you pose a word problem to learners, you should:
  - Ensure the learners understand the necessary mathematical vocabulary and skills;
  - Encourage learners to think creatively;
  - Assist learners in developing their own methods of calculation;
  - Provide real-life contexts that help the learners to generalise mathematical knowledge

Steps to follow when teaching word problems to Foundation Phase learners:

- You can give learners multiple problems. Encourage a discussion on the information that is known and what needs to be found out.
- Analyse the problem by reading the problem with understanding;
- Help learners understand the specific mathematical vocabulary and skills stated in the problem and explain unfamiliar words.
- Determine the basic operations needed to solve the problem by asking higher order questions:
  - What information is given?
  - What is the question that needs to be answered?
  - Can you re-write the question in your own words?
  - Can you draw a sketch or picture to illustrate the question?

- What kind of problem is it: addition, subtraction, multiplication or division?

- Learners should use the following strategies for solving the problems:
  - Using concrete apparatus;
  - Drawing pictures, diagrams or sketches;
  - Building up and breaking down numbers;
  - Doubling and halving numbers;
  - Using a number line;
  - Making lists or tables;
  - Guessing and checking different solutions;
  - Working backwards and forwards;
  - Working from the known to unknown.

- Learners should be exposed to different word problems that represent repeated addition. Repeated addition is done when the same number is repeatedly added. Repeated addition word problems can also be addressed using money, mass, length and capacity.

Applying strategies to solve a problem

Example 1

Repeated addition using problem solving strategies.

If Sam reads 34 books each month, how many books would she have read altogether in July, August and September?

Problem solving steps:

Step 1: Know or understand the problem

- Help learners to understand the information given and to determine exactly what they are expected to do:
- Ask learners to identify the information that is given;
- Ask learners to underline the words in the question that give the information.

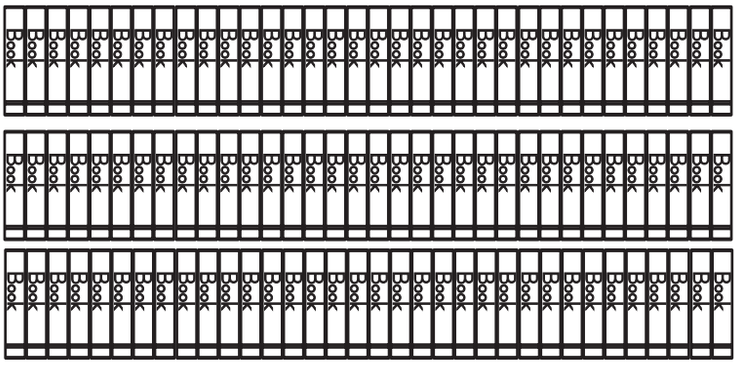
If Sam reads 34 books each month how many books would she have read altogether in July, August and September?

- Help learners identify what is given by asking appropriate questions.
  - How many books does Sam read? 34 books.
  - When does she read the books? Each month.
  - How many months are given? 3 months: July, August and September.
  - What does the question ask? How many books does Sam read altogether in 3 months?

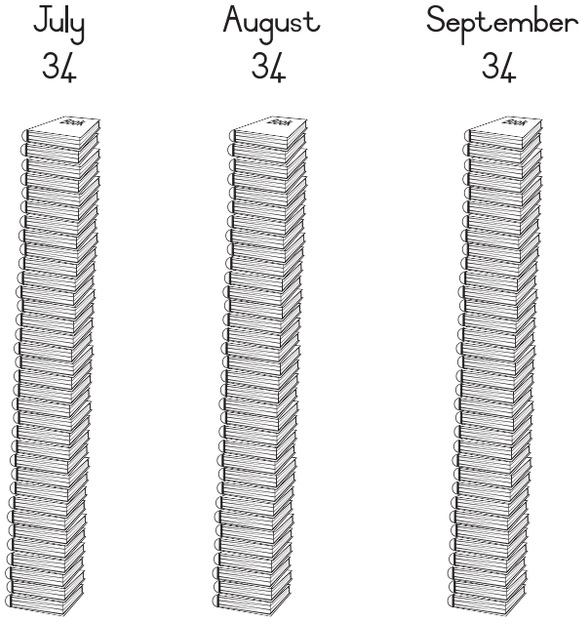
Step 2: Design a plan

- Help learners think about what needs to be done to find the solution.
- Determine with the learners what operations to use and then where to start and which steps to follow.
- Learners can draw a diagram to help them understand the question or write a number sentence, depending on their level of understanding, for example:

A drawing of 3 groups of 34 books each. (It is unlikely that learners would draw something this detailed – this would waste a lot of time. Rather encourage the learners to use numbers and number sentences to find the solution.)



34 + 34 + 34 = _____



3 groups of 34 books

3 x 34 = _____

Step 3: Carry out your plan

- Solve the problem using the strategy selected. Learners will select different strategies to solve the problem:

Repeated addition using breaking down and building up as a strategy:	Repeated addition using multiplication as a strategy:
34 + 34 + 34 = _____	34 + 34 + 34 = 3 x 34
= (30 + 4) + (30 + 4) + (30 + 4)	= (3 x 30) + (3 x 4)
= (30 + 30 + 30) + (4 + 4 + 4)	= 90 + 12
= 90 + 12	= (90 + 10) +2
= 90+ 10 + 2	= 100 + 2
= 102	= 102

Step 4: Look back

- Check the solution to the problem with the learners.
- The learners should make sure that the calculation was done correctly and the strategy was applied correctly.
- Ask learners to read the question again and to write the correct answer:

  If Sam reads 34 books each month how many books would she have read altogether in July, August and September?
  Sam would have read 102 books in 3 months.

Example 2

- Repeated addition using breaking down and building up as a strategy.

  Mpho helped her mum for 5 days to bake cupcakes.
  They baked 28 cupcakes every day. How many cupcakes did they bake altogether?

Problem solving steps:

Step1: Know or understand the problem

- Help learners to understand the information given and to determine exactly what they are expected

to do:

- Learners should identify the information that is given;
- Learners should underline the words in the question that give the information.

Mpho helped her mum for 5 days to bake cupcakes.
They baked 28 cupcakes every day. How many cupcakes did they bake altogether?

- Help learners identify what is given by asking appropriate questions.
  - What did Mpho and her mum bake? (Cupcakes)
  - For how many days did Mpho help her mum? (5 days)
  - How many cupcakes did they bake each day? (28 cup cakes per day)
  - What does the question ask? (How many cupcakes did they bake altogether?)
  - What should you do? (Add all the cupcakes together for 5 days)

Step 2: Design a plan:

- Help learners to think about what they have to do. Help them to determine which operations to use, where to start with the problem and which steps to follow.
- Ask learners to draw the question to make it meaningful or to write a number sentence, depending on their level of understanding, for example:

5 groups of 28 cupcakes

28 + 28 + 28 + 28 + 28 = _____

- The numbers are too large to add up by counting on in multiples and should be broken down to add correctly.

Step 3: Carry out your plan

- Solve the problem using breaking down and building up as a strategy.

$28 + 28 + 28 + 28 + 28 =$ _____
$= (20 + 8) + (20 + 8) + (20 + 8) + (20 + 8) + (20 + 8)$
$= (20 + 20 + 20 + 20 + 20) + (8 + 8 + 8 + 8 + 8)$
$= 100 + (8 + 8) + (8 + 8) + (8)$
$= 100 + 16 + 16 + 8$
$= 100 + 10 + 10 + 6 + 6 + 8$
$= 120 + 12 + 8$
$= 120 + 10 + 2 + 8$
$= 120 + 20$
$= 140$

Step 4: Look back

- Check the solution to the problem with the learners.
- The learners should make sure that the calculation was done correctly and the strategy applied correctly.
- Learners could use other strategies and methods to calculate the answer. Encourage them to find the most efficient calculation strategies.
- Ask the learners to read the question again and to write the full answer:

Mpho helped her mum for 5 days to bake cupcakes.
They baked 28 cupcakes every day. How many cupcakes did they bake altogether?

Mpho and her mum baked 140 cupcakes in 5 days.

Example 3

- Repeated addition using multiplication as a strategy.

Jonny cleaned the garage. His dad paid him R30 per day. How much money did Jonny earn for the work he did in 4 days?

Skills needed to solve the problem:

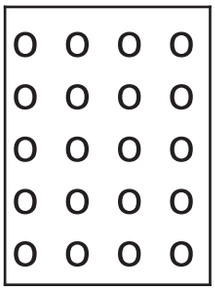
- Multiplication of whole numbers:
- Learners should be able to multiply 2 digits by 1 digit.
  - Ensure that learners do not initially learn multiplication tables off by heart but move from skip

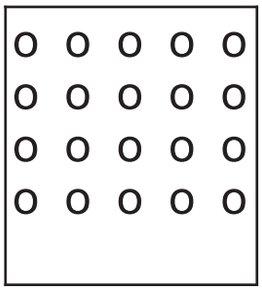
counting and repeated addition to seeing the patterns in multiplication tables up to 10 x 10.

- Ask learners if they see a pattern when counting in 4s from 0 – 40.
- Ask learners if they see a pattern when counting in 5s from 0 – 50.
- Ask learners if they see a pattern when counting in 2s from 0 – 40 and in 4s from 0 - 40.
- Ask learners if they see a pattern when counting in 3s from 0 – 60 and in 6s from 0 - 60.
- It will be highly beneficial to learners if they eventually do know their multiplications tables off by heart.

- Commutative property:
  - Learners should know that numbers can be multiplied in any order e.g. 5 x 6 = 6 x 5

Ask learners to draw the following arrays of counters on their slates or whiteboards or in their maths books:

1. 

2. 

- Ask learners to write a number sentence for each to calculate how many circles there are in each rectangle.

1. $5 \times 4 = 20$ 2. $4 \times 5 = 20$

- Learners should notice that you get the same answer when you multiply the same two numbers, no matter what order you do it in. It does not matter in which order the numbers are written.

Multiplication of units by 10:

- Ask learners to write down and calculate the following:

$4 \times 7 =$
$40 \times 7 =$

- Ask the learners what they notice.
- Do more similar sums with the learners and ask them what they notice, e.g. 6 x 5, 60 x 5; 3 x 9, 3 x 90.

Multiplication of 2 digits by 1 digit using breaking down and building up:

- Ask learners to calculate 16 x 4 by breaking up 16 on a slate or whiteboard or in a maths book.
  - Learners write 16 in expanded notation:

    $16 = 10 + 6$
- Encourage learners to multiply both numbers by 4 and to add the answers.

Example:
$$16 \times 4 = (10 + 6) \times 4$$
$$= (10 \times 4) + (6 \times 4)$$
$$= 40 + 24$$
$$= 40 + 20 + 4$$
$$= 64$$

Problem solving steps:

Step 1: Know or understand the problem

- Help learners to understand the information given and to determine exactly what they are expected to do:
- Learners should identify the information that is given.
- Learners should underline the words in the question that give the information.

Jonny cleaned the garage. His dad paid him R30 per day.
How much money did Jonny earn for the work he has done in 4 days?

- Learners should identify what is given. Help them to do this by asking appropriate questions:
  - What did Jonny do? Clean the garage.
  - How many days did it take him? 4 days.
  - How much money did his dad pay him? R30 per day.
  - How much money did he earn?

Step 2: Design a plan

- Help learners think about what they have to do. Help them determine which operations to use, where to start with the problem and which steps to follow.
- Ask learners to draw the question to make it meaningful or to write a number sentence, depending on their level of understanding, for example:

4 groups of R30

R30 + R30 + R30 + R30 = _____

- Learners may realise that a shorter way to write the number sentence is:

$$4 \times 30 =$$

Step 3: Carry out your plan:

- Solve the problem using multiplication as a strategy.

$$4 \times 30 \text{ or } 30 \times 4$$

- Encourage learners to break up numbers to make them easier to multiply:
  - Ask learners to calculate: 4 x 30
  - Ask them to break up the 30 and then multiply:

$$4 \times (3 \times 10) = (4 \times 3) \times 10 = 12 \times 10 = 120$$

- Ask learners what they notice.

Step 4: Look back:

- Check the solution to the problem with the learners.
- The learners should make sure that the calculation was done correctly and the strategy applied correctly.
- Ask learners to read the question again and write the full answer:

Jonny cleaned the garage. His dad paid him R30 per day.
How much money did Jonny earn for the work he has done in 4 days?

Jonny earned R120 for 4 days work.

Another example of how multiplication can be tested

ANA 2014 Grade 3 Mathematics Item 18.2

18.2 $5 \times 10 =$ _____

Notes:

Word problems: division

ANA 2013 Grade 3 Mathematics Item 20

20. There are 36 chocolates in a box. Each child gets 5 chocolates. How many children got chocolates? How many chocolates were left over?

What should a learner know to answer this question correctly?

Learners should be able to:

- Read the problem with understanding;
- Analyse the problem to determine what information is known and what information is still needed to solve the problem;
- Select the most appropriate strategy to share - draw pictures or do division;
- Understand the difference between equal grouping and sharing;
- Understand equal sharing and division with remainders;
- Understand basic mathematical vocabulary such as: how many, each, left over, equal sharing, share amongst, division, remainder, and know the spelling of the word 'remainder' in their home language.

Where is this topic located in the curriculum? Grade 3 Term 1

Content area: Numbers, Operations and Relationships.

Topic: Grouping and sharing leading to division.

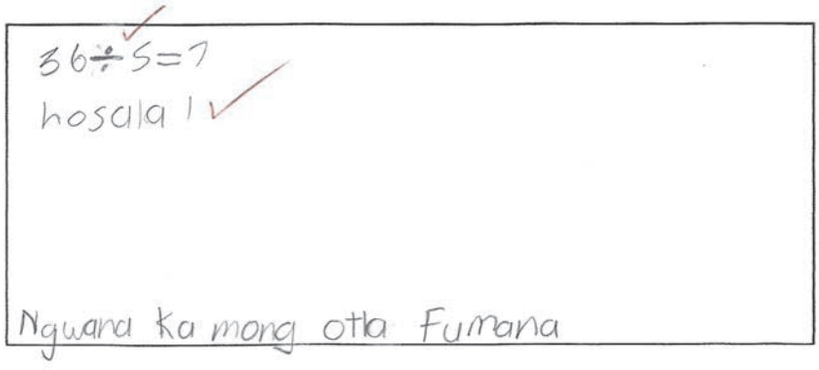
Concepts and skills:

- Solves and explains own solutions to problems that involve equal sharing and grouping up to 50 with answers that may include remainders.

What would show evidence of full understanding?

- If the learner gave the answer '7 children, remainder 1': this shows the learner was able to identify the basic operation that had to be applied, namely, division of a two-digit number.
- The learner was able to apply division correctly (36 ÷ 5 = 7).
- The learner could write the remainder correctly as 1 and could correctly write the word 'remainder' in his/her home language.

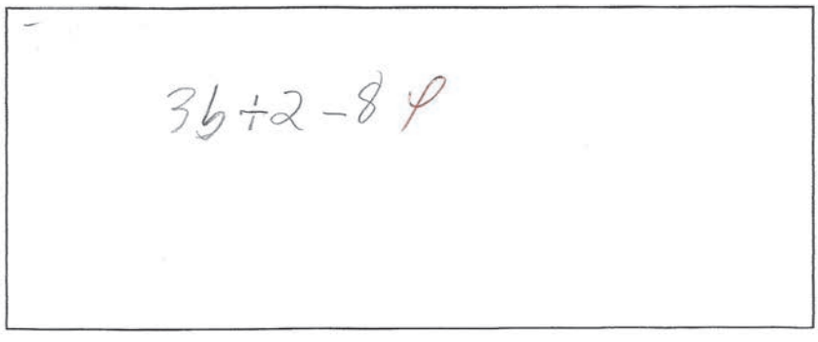
20. Ho ne ho na le dit jhokolete tse 36 ka hara lebokose. Ngwana ka mong o ne a filwe tse 5. Ke bana ba bakae ba fumaneng dit jhokolete mme ho setse tse kae?



What would show evidence of partial understanding?

- If the learner followed the correct basic operation of division, but did not use the correct divider, 5.

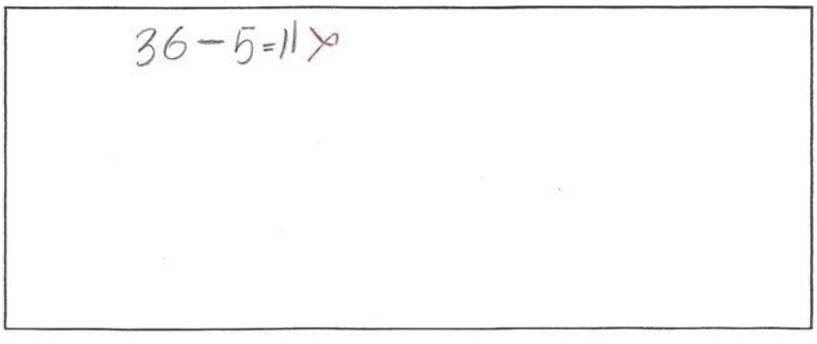
20. Ho ne ho na le dit jhokolete tse 36 ka hara lebokose. Ngwana ka mong o ne a filwe tse 5. Ke bana ba bakae ba fumaneng dit jhokolete mme ho setse tse kae?



What would show evidence of no understanding?

- If the learner gave the wrong answer and used the incorrect method or basic operation to solve the problem.

20. Ho ne ho na le dit jhokolete tse 36 ka hara lebokose. Ngwana ka mong o ne a filwe tse 5. Ke bana ba bakae ba fumaneng dit jhokolete mme ho setse tse kae?



What do the item statistics tell us?

20% of learners answered the question correctly.

Factors contributing to the difficulty of the item

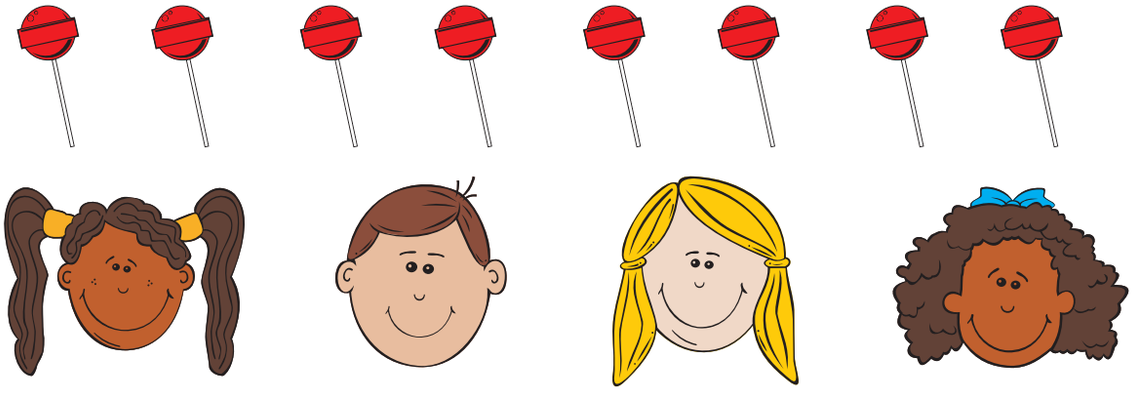
- The problem was difficult for learners who struggled to read and understand the question. Learners may not have been able to determine the basic operation needed to solve the problem.
- Although CAPS requires that in Term 1 in Grade 3 learners should be able to solve and explain solutions to practical problems that involve equal sharing and grouping up to 50 with answers that include remainders, learners could not determine the answer, even practically.
- It is evident that learners were taught division, but were not able to do the calculation of division correctly as yet.

Teaching strategies

Solving of problems using equal sharing and grouping with no remainder

- In order for learners to correctly solve this item teachers should emphasise the skills required by giving learners many examples of word problems to solve. Word problems should be done during the process of learning about operations and not only at the end.
- Learners need to understand the two types of sharing situations:
  - Equal sharing;
  - Equal grouping.
- Equal grouping and sharing vocabulary: Pieces, parts, share/s, group/s, share equally, share amongst, the same as, equal shares or parts, more or less than, break up, each, per, divide.
- To help learners understand sharing and grouping, expose them to practical sharing situations that arise in various contexts in their lives, e.g. food, sweets, pencils, money, length, capacity, mass.
- Equal sharing and grouping should be introduced by beginning with small numbers (one-digit numbers) and increasing the number range to two-digit numbers.
- Share objects practically between two learners at first and then between more than two learners, e.g. between 3, 4, 5, 6, 7, 8 and 9 learners.
- Encourage discussion and allow learners to find their own ways to solve the problems by allowing learners to draw the problems or to demonstrate them practically with concrete resources.

Examples to demonstrate the difference between sharing and grouping



Example of a sharing problem

We share 8 sweets equally amongst four children.
How many sweets does each child get?

Each child gets 2 sweets.

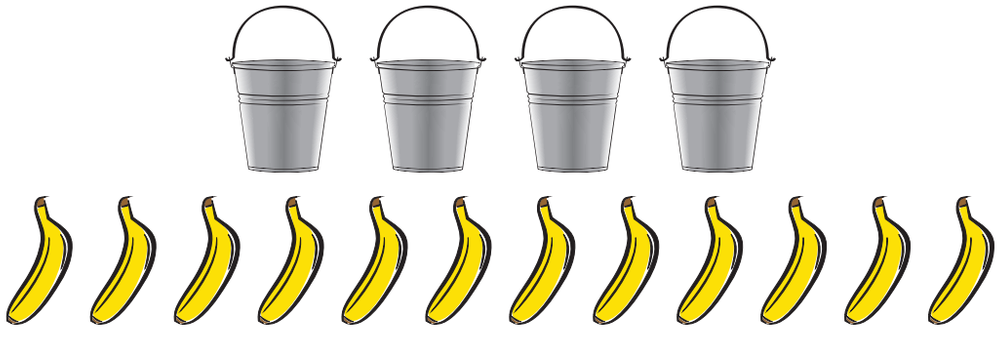
Example of a grouping problem

We have 8 sweets.
How many children will each get four sweets?

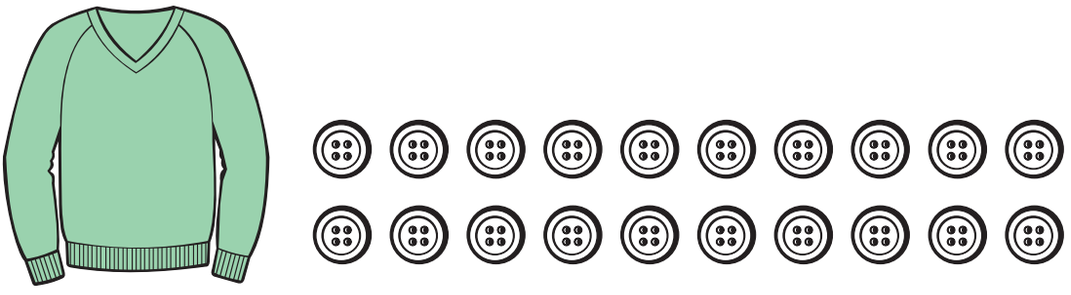
Two children can get 4 sweets each.

Grouping and Sharing – two more problems to work though in class

1. A girl has 12 bananas. She puts the same number of bananas in each bucket. How many bananas will she put in each bucket?



2. Pam had 20 buttons for all the jerseys she knitted. She used 5 buttons for each jersey. How many jerseys did she knit?



Concrete activity solution

- Ask learners to put the buttons into groups of 5.
- Ask learners to estimate the result of the problem before actually doing the activity.
- Encourage learners to find their own way to share or group objects.
- Ask learners to explain and record their methods.
- Remember that learners should always start to equally share and group at a concrete level and then proceed to the semi-concrete and the abstract level.

Solving of problems using equal sharing and grouping with remainders

- Learners should first be exposed to problems without remainders and then problems with remainders.
  - **Practical** work is essential to ensure understanding of equal sharing or grouping. Learners will have to work with increasingly larger numbers within the number range 0-100.
  - **Remember: L**earners should use their own techniques when solving equal sharing problems.
  - Encourage learners to estimate the result of the problem before actually doing the activity.

- Steps for dealing with a remainder:
  - Learners should equally share or group objects.
  - If there is a remainder, learners should discuss what to do with the remainder.
  - Mathematical vocabulary that learners should be taught is: equally share, share amongst, share between, divide, remain, remainder, left over, grouping.

Practical group work activity:

- Ask learners to work in groups of 2.
- Ask learners to share 16 counters equally between the two learners in each pair.
  - Ask how many does each learner get? Answer: 8

- Ask learners to share 18 counters equally between the two learners.
  - Ask how many does each learner get? Answer: 9

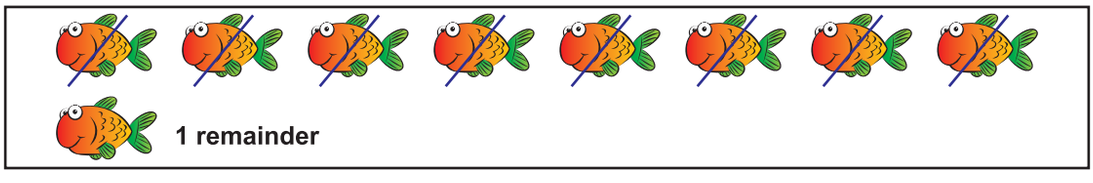
- Now divide different numbers of objects into different sized groups, for example, divide 12 counters into groups of 5;15 counters into groups of 2; 21 counters into groups of 4 and 24 counters into groups of 5, making sure that the numbers used will have remainders when the division has been done.
  - Discuss the answers with the learners and check the way in which the learners did the practical sharing and grouping while they worked in their pairs.
  - Encourage discussion and encourage learners to find their own ways to solve the problems.

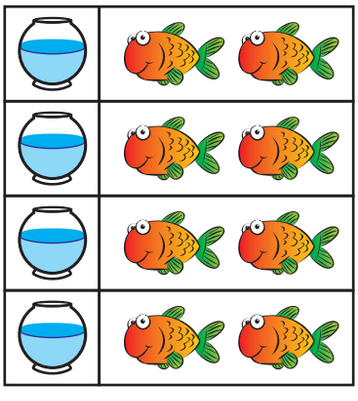
Whole class discussion:

Explain to learners how to share objects equally by going through the following examples:

1. Share 9 fish equally amongst 4 fish bowls. How many fish will be in each fish bowl?

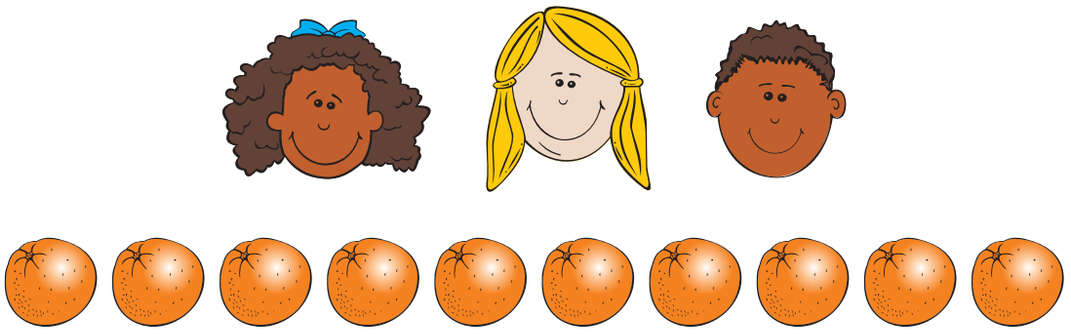
- Draw pictures on the board similar to those below to demonstrate the grouping of the fish into the bowls.





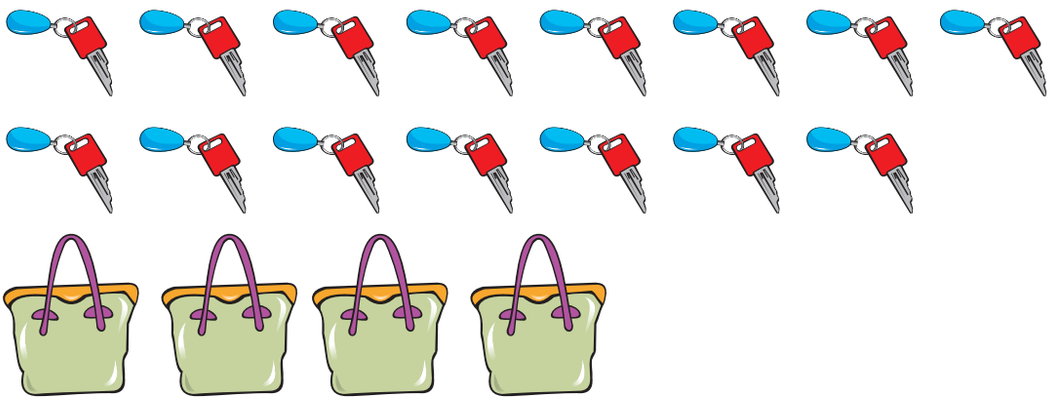
- Cross out each fish once you have shared it.
- You can draw a dot instead of a fish as pictures take quite long to draw.
- Learners must be able to write: 'Each fish bowl will have 2 fish and 1 fish will be left over'.
- Explain to learners that the left over fish is called a remainder.

2. Share 10 oranges equally amongst 3 children. How many oranges will each child get? How many oranges will be left over?



3. Share 15 flowers amongst 2 vases. How many flowers will each vase get? How many flowers will be left over?

4. Share 15 keys amongst 4 handbags. How many keys will be put in each handbag? How many keys will not be in a handbag?



Solving of problems using division

- As learners grow in confidence with finding their own ways to share and group, gradually introduce the appropriate mathematical term 'divide' and the symbol '÷' that will help them to use shorter ways to explain and represent their ideas.
- Give learners many different opportunities to develop their understanding of using the '÷' symbol and division number sentences to describe both grouping and sharing problems.

- For example, ask learners to write a quick mathematical way of finding the answer to grouping and sharing problems.
- Learners should understand that they can write **30 ÷ 5 = 6** for each of the following problems:

- Grouping
  How many packets of sweets can I make if I have 30 sweets and I put 5 sweets in each packet? (30 ÷ 5 = 6)

- Sharing
  Mother shares R30 equally amongst 5 children. How much money does each child get? (R30 ÷ 5 = R6)

Division as equal sharing

- In a sharing situation, a known quantity of items (e.g. balloons, balls and buttons) is shared equally amongst a known number of entities (e.g. people, packages, boxes). What is not known in a sharing situation is the number of the given item per share.

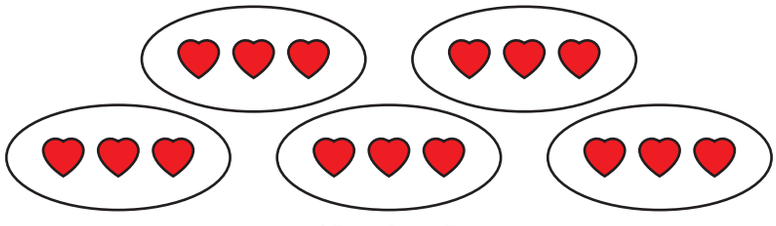
- In the following picture 15 ÷ 3 = 5 is the number of the balloons each teddy bear gets if 15 balloons are shared equally amongst 3 teddy bears.

15 ÷ 3 = 5

Division as equal grouping

- In a grouping situation a given quantity of items is shared between an unknown number of groups of a given size. What is not known in a grouping situation is the number of groups.

- In the following picture 15 ÷ 3 = 5 is the number of equal groups of 3 hearts you can make with 15 hearts.



15 ÷ 3 = 5

- Consolidation: Use the printable worksheets to give your learners lots of opportunities to do division by sharing and grouping **(see printables).**

- **Remember: L**earners should be encouraged to use their own techniques when solving equal sharing problems.

Other examples of how division can be tested

ANA 2014 Grade 3 Mathematics Item 19

19. Mum shared 42 sweets equally amongst her 3 children.

How many sweets did each child get?

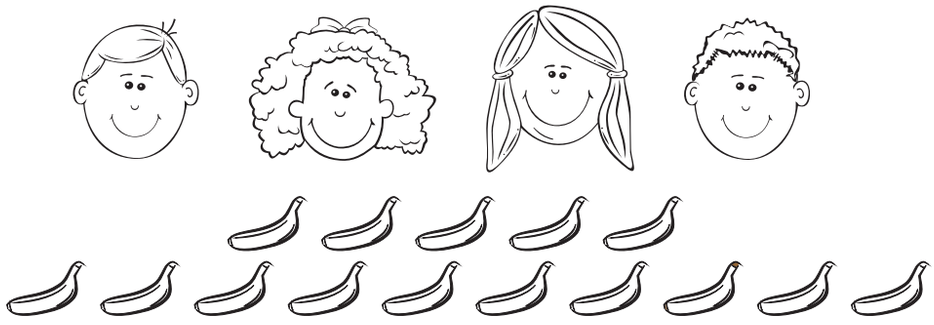
ANA 2014 Grade 3 Mathematics Item 26

26. Calculate 42 ÷ 2.

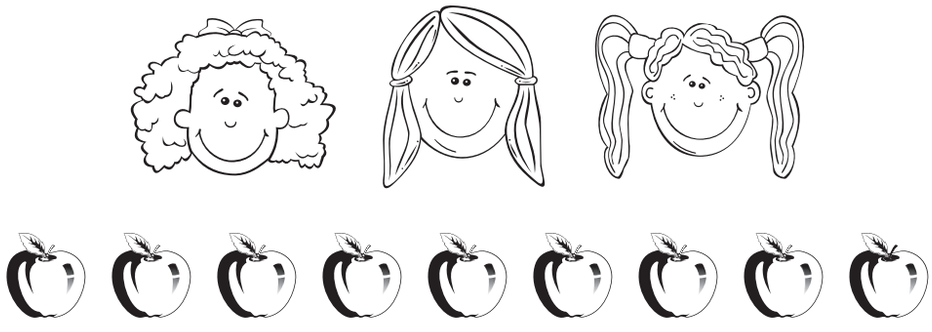
Notes:

Printable: Sharing (1)

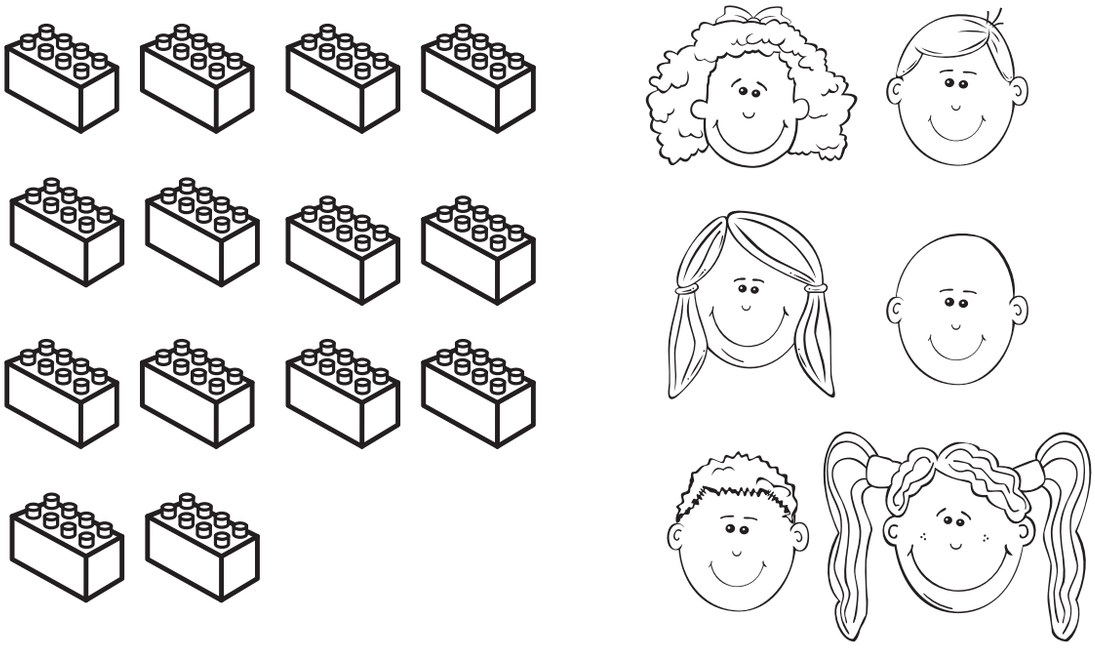
Share equally each time. Will there be a remainder?



How many bananas will each child get? _____

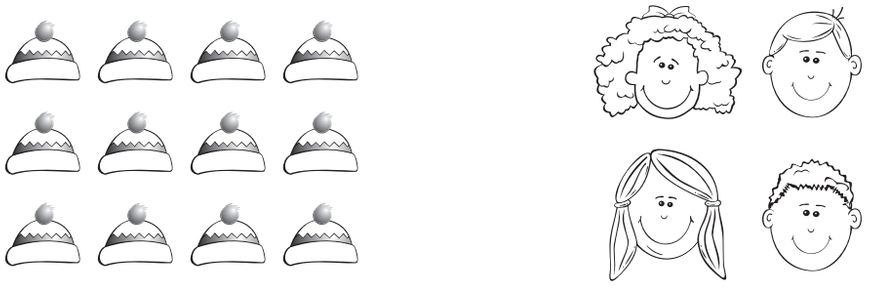


How many apples will each child get? _____

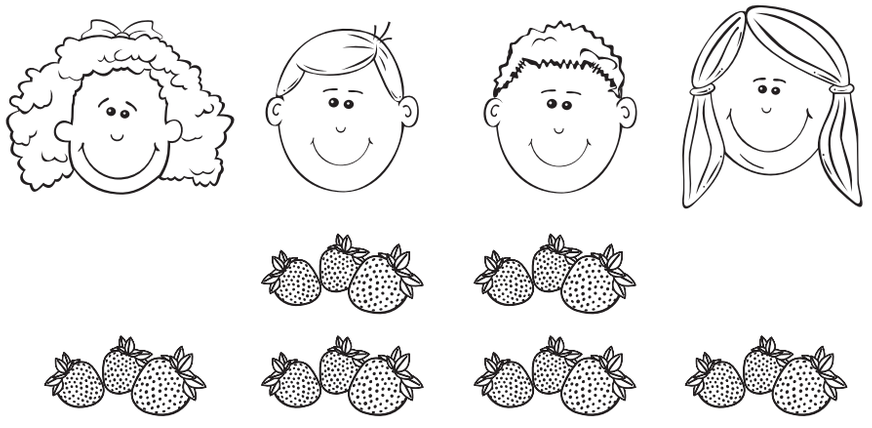


How many blocks will each child get? _____

Printable: Sharing (2)

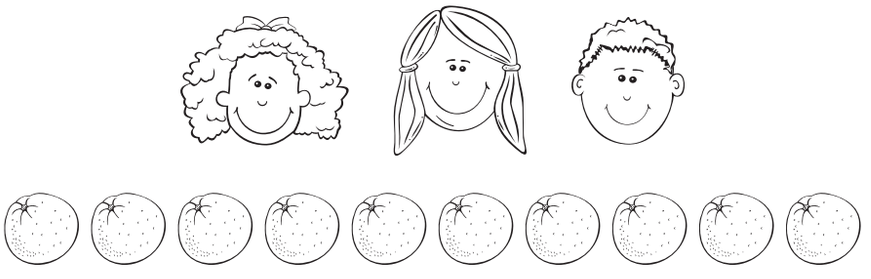


How many hats will each child get? _____



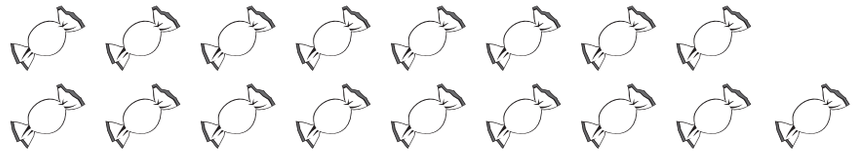
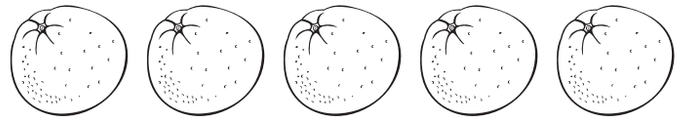
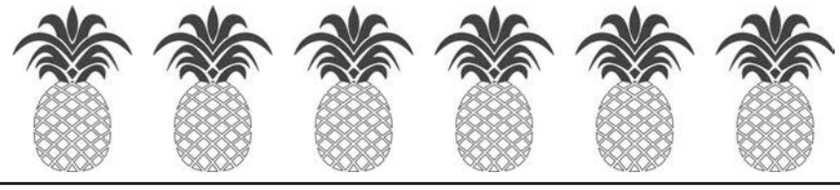
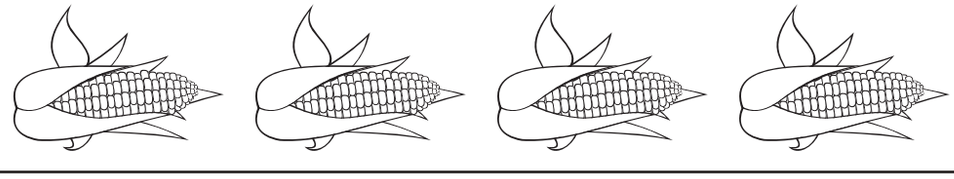
How many strawberries will each child get? _____

How many bows will each child get? _____



How many oranges will each child get? _____

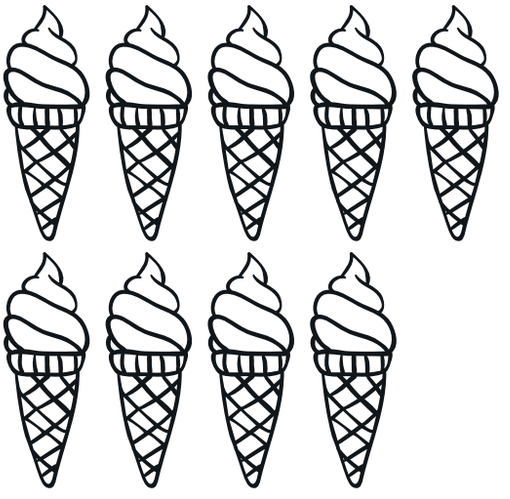
Printable: Sharing (3)

Share equally between two children: How many will each child get? Will there be any left overs? How many?	
	
	
	
	
	
	
	
	
	

Printable: Grouping

Put into groups of 3. How many groups will you make?
Are there any that are left over after you grouped the ice creams? How many?

9



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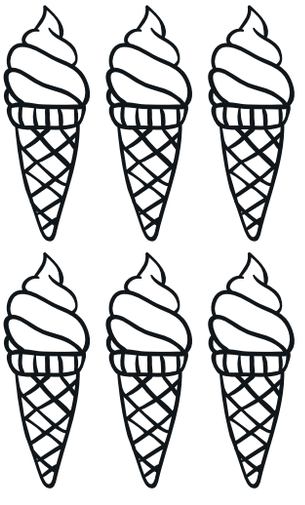

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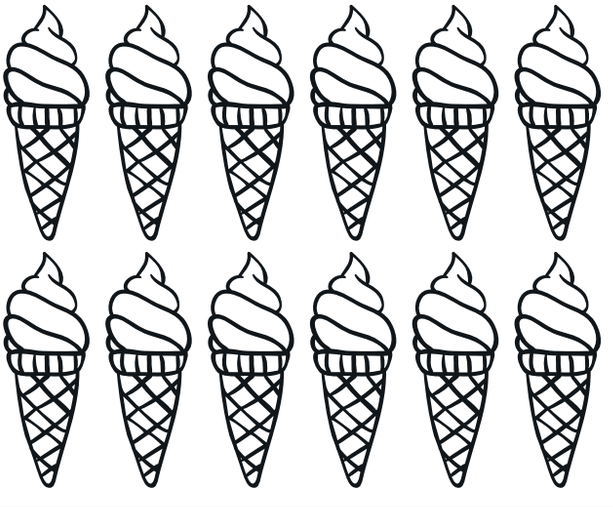


Numeric patterns: counting forwards and backwards

ANA 2013 Grade 3 Mathematics Item 10.1 and 10.2

10. Complete the table:

10.1	Count forwards in 100s	584				
10.2	Count backwards in 20s	320				240

What should a learner know to answer these questions correctly?

Learners should be able to:

Item 10.1

- Understand the mathematical vocabulary of counting forward in 100s;

Item 10.2

- Understand mathematical vocabulary of counting backwards in 20s;
- Complete a specific number sequence by counting forwards or backwards.

Where is this topic located in the curriculum? Grade 3 Term 1 - 3

Content area: Numbers, Operations and Relationships.

Topic: Count forwards and backwards.

Concepts and skills:

- Count forwards and backwards in 20s, 25s, 50s and 100s to at least 1000.

What would show evidence of full understanding?

Item 10.1

- If the learner completed the number sequence correctly this shows a full understanding of counting forwards in 100s.

Item 10.2

- If the learners completed the number sequence correctly this shows a full understanding of counting backwards in 20s.

10. Complete the table:

10.1	Count forwards in 100s	584	684	784	884	984
10.2	Count backwards in 20s	320	300	280	260	240

What would show evidence of partial understanding?

Item 10.1

- If the learner counted forwards in 1s instead of 100s, the learner shows an understanding of counting forwards, but no understanding of the place value of digits in numbers.

Item 10.2

- If the learner counted forwards in 100s in both questions it shows that the learner did not read the instructions with understanding.
- If the learner counted forwards or backwards incorrectly, the learner showed some understanding of counting forwards and backwards but could not count in the correct multiple of 100 or 20.

10. Complete the table:

10.1	Count forwards in 100s	584	684	784	884	984
10.2	Count backwards in 20s	320	420	520	620	240

10. Complete the table:

10.1	Count forwards in 100s	584	585	586	587	588
10.2	Count backwards in 20s	320	310	320	330	240

10. Complete the table:

10.1	Count forwards in 100s	584	585	586	587	588
10.2	Count backwards in 20s	320	321	322	323	240

What would show evidence of no understanding?

- No response to the question; or
- If the learner wrote a random number that makes no sense.

10. Complete the table:

10.1	Count forwards in 100s	584	4	5	8	100
10.2	Count backwards in 20s	320	3	2	0	240

What do the item statistics tell us?

Item 10.1

37 % of learners answered the question correctly.

Item 10.2

30% of learners answered the question correctly.

Factors contributing to the difficulty of the items

Item 10.1

- The concept and skills tested in this item, such as counting forwards in 100s have not been mastered by majority of learners.
- Learners find it difficult to understand which digit in a 3-digit number must increase when counting in 100s.

Item 10.2

- The concept and skills tested in this item, such as counting backwards in 20s have not been

mastered by majority of learners.

- Although learners are familiar with numbers in tables, the two questions posed together in one table could have been confusing to learners in Grade 3 due to poor reading skills.

Teaching strategies

Counting forward and backwards in 20s and 100s using number cards

- Help the learners to realise that counting in intervals is a quick way of counting and that this strategy can help them in all number operations.

Concrete Level

- If learners struggle with counting in multiples, go back to Grade 2 activities and reinforce on all the different levels if necessary.

Abstract Level

- Give learners cards with numbers in multiples and place them according to number in ascending or descending order.
- Colour the multiples on the number chart and count in 100s or 20s forwards and backwards.
- Encourage learners to move towards counting in multiples using the number chart without colours, counting forwards and backwards in 100s and 20s.

Counting using a 100 chart/number chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- Ask learners to complete a variety of activities to reinforce their counting skills, such as:
  - Connect the dots to complete pictures by counting in multiples;
  - Use counting forwards and backwards as a strategy for solving word problems involving equal sharing and grouping, addition and subtraction, money and time;

- Ask learners to explain their solutions.
- Let learners check their peers' solutions to problems.

- Discover, describe and copy a pattern of counting in multiples.
- Ask questions about the position of multiples, e.g. “Which multiple of 10 comes after 37? (40) or before 45? (40) or between 67 and 73?” (70).
- Fill in the missing numbers on the number line or blank number chart.
- Gradually encourage learners to work towards counting without a number chart or number line (counting mentally only).

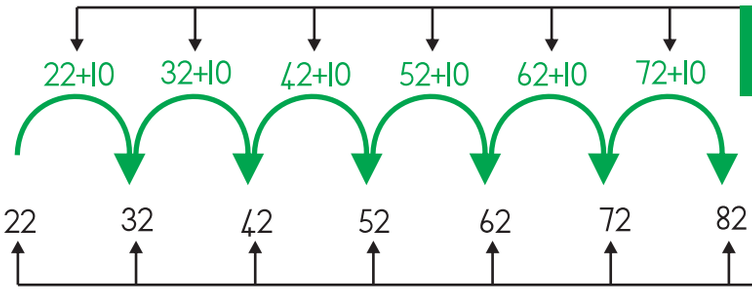
- Remember to also work in a higher number range (for example: 500-600)

501	502	503	504	505	506	507	508	509	510
511	512	513	514	515	516	517	518	519	520
521	522	523	524	525	526	527	528	529	530
531	532	533	534	535	536	537	538	539	540
541	542	543	544	545	546	547	548	549	550
551	552	553	554	555	556	557	558	559	560
561	562	563	564	565	566	567	568	569	570
571	572	573	574	575	576	577	578	579	580
581	582	583	584	585	586	587	588	589	590
591	592	593	594	595	596	597	598	599	600

Counting forward and backwards in 20s and 100s using number lines

- Number lines should be used to teach counting forwards and backwards in 10s and 100s.
- Learners must understand that counting in 10s and 100s is not the same. If they count in tens the tens' digit increases and if they count in hundreds the hundreds' digit increases.

- Count in 10s on a number line:
  - The patterns in all three examples are the same: the number in **the tens place goes up by one each time.**

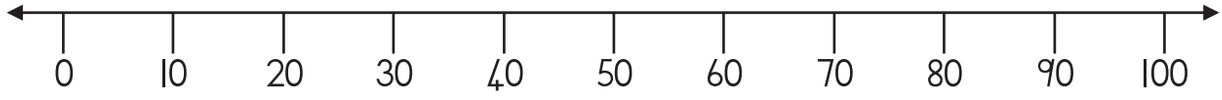


Add 10 to get the next number when skip counting by 10s.

Can you see a pattern? The digit in the tens place is increasing by 1.

- Ask the learners to complete a variety of examples, such as the following:
  - Draw these number lines on the board.
  - Use them to count in tens.

a).



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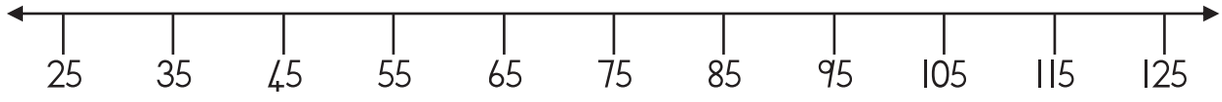

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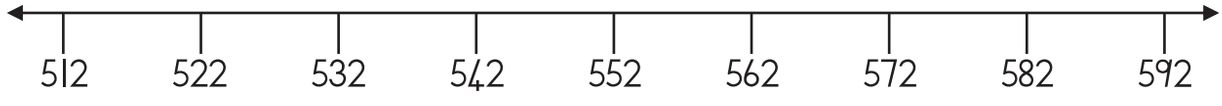
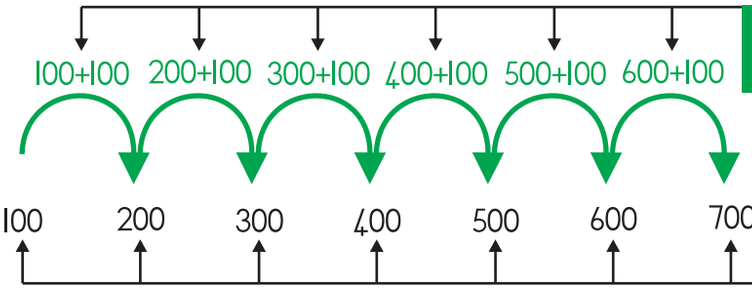


- Count in 100s on a number line:
  - The three examples below show counting in hundreds.
  - The patterns in all three are the same: the number in the hundreds place goes up by one each time.

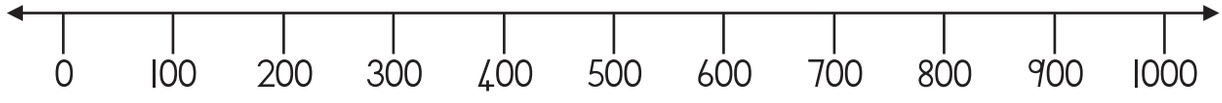


Add 100 to get the next number when skip counting by 100s.

Can you see a pattern? The digit in the hundreds place is increasing by 1.

- Ask the learners to complete a variety of examples, as indicated below:
  - Draw these number lines on the board.
  - Use them to count in hundreds.

a).



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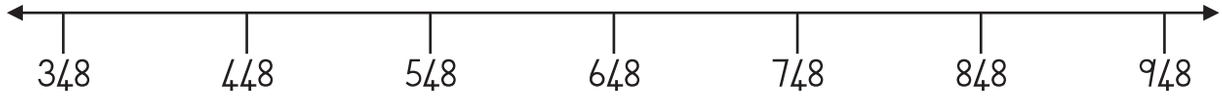
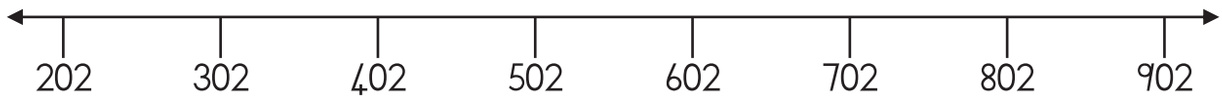


c).



Other examples of how counting forwards and backwards can be tested

ANA 2014 Grade 3 Mathematics Item 3

2. Count backwards in 100s from 521 to 121.

521; _____; _____; _____; 121

ANA 2014 Grade 3 Mathematics Item 7.2

7.2 Count forwards in 20s.

220; 240; _____; _____; _____

Notes:

Numeric patterns

ANA 2013 Grade 3 Mathematics Item 15

15. Write the next number in the number sequence below.

125; 175; 225; 275; _____

What should a learner know to answer this question correctly?

Learners should be able to:

- Understand the mathematical vocabulary relating to number patterns, e.g. next number and number sequence;
- Identify a specific number sequence by applying subtraction to determine the specific number pattern and then add that number to complete the number sequence.

Where is this topic located in the curriculum? Grade 3 Term 1 - 3

Content area: Patterns, Functions and Algebra.

Topic: Number patterns.

Concepts and skills:

- Copy, extend and describe simple number sequences to at least 1000.

What would show evidence of full understanding?

- If the learner completed the specific number sequence by increasing the numbers by 50.

15. Write the next number in the number sequence below.

125; 175; 225; 275; 325 ✓

What would show evidence of partial understanding?

- If the learner decreased the numbers by 50 instead of increasing the numbers by 50; or
- If the learner decreased the numbers by twenty: this shows very little understanding.

15. Write the next number in the number sequence below.

125; 175; 225; 275; 255 ✗

What would show evidence of no understanding?

- No response to the question; or
- If the learner wrote a random number that makes no sense.

15. Write the next number in the number sequence below.

125; 175; 225; 275; 505

What do the item statistics tell us?

35% of learners answered the question correctly.

Factors contributing to the difficulty of the item:

- Concepts and skills tested in this item, such as number patterns, may not have been mastered by the majority of learners.
- Learners could not correctly identify the correct number sequence as they may not have known how to apply strategies to determine the correct number pattern.

Teaching strategies

Copy, extend and describe simple number patterns.

- It is important that learners are able to sequence numbers by counting forwards and backwards in:
  - Ones, from any number between 0 and 1000;
  - Twos, from any multiple of 2 between 0 and 200;
  - Threes, from any multiple of 3 between 0 and 200;
  - Fours, from any multiple of 4 between 0 and 200;
  - Fives, from any multiple of 5 between 0 and 200;
  - Tens, from any multiple of 10 between 0 and 200;
  - Even and odd numbers up to 200.

Revision of basic patterns

- Revise all number patterns in multiples of 2, 3, 4, 5 and 10 from any given number before allowing learners to count in 50s;
- Encourage learners to identify a number sequence on their own by looking at how the number values of the digits change as the numbers increase or decrease in size.

- Show learners that a number pattern can be identified by subtracting the first two or three numbers in the pattern from each other.

  Example:

  125, 175, 225, 275, _____

$175 - 125 = 50,\ 225 - 175 = 50$

- The number pattern is that the number increases by 50 each time.
- Thus learners should add 50 to 275 to get the next number, 325.

- Give learners enough opportunity to work with different kinds of number patterns.
- Here are some examples of activities to copy, extend and describe simple number patterns:
- Write down these numbers on the board. Ask who can see which pattern these numbers belong to?

410, 412, 414, 416, 418, 410
Answer: 2s pattern

65, 70, 75, 80, 85, 90
Answer: 5s pattern

410, 420, 430, 440, 450, 460
Answer: 10s pattern

- Ask learners to tell you which number does not belong to the pattern and to give reasons for their answers.

57, 27, 87, 67, 72 _____
Answer: 72: it does not end in a 7

29, 67, 28, 25, 24 _____
Answer: 67: the tens digit is not 2

15, 52, 20, 30, 45 _____
Answer: 52: it does not belong to the 5s pattern

12, 24, 30, 19, 27 _____
Answer: 19: it does not belong to the 3s pattern

416, 614, 164, 616 _____
Answer: 616: the only number that does not have 4 as a digit

- Write or put numbers on the board using chalk or number cards.
  - Ask learners to look at the numbers.
  - Tell the learners that the numbers may not be in order and that all the numbers in the sequence

might not be there.

- Ask learners to tell you which number does not belong to the pattern.
- Ask learners to give you their reasons for their answers.

65, 64, 70, 55, 60, 50

Answer: 64 does not belong to the 5s pattern

210, 220, 203, 240, 250, 260

Answer: 203 does not belong to the 10s pattern

765, 770, 785, 797, 810

Answer: 797 does not belong to the 5s pattern

540, 250, 580, 130, 755

Answer: 755 does not belong to the 10s pattern

402, 401, 426, 438, 424, 310

Answer: 401 does not belong to the 2s pattern

Twos patterns

- Write the following numbers on the board

221, 219, 217, _____, _____, _____,

- Discuss the pattern using the following questions:
- Which pattern do the numbers belong to? The increasing in 2s pattern.
- Are the numbers that are missing on the left or right? On the right.
- So are the missing numbers going to be bigger or smaller than 217? Smaller, because the number sequence is decreasing by two.
- Use your 2s pattern to work out what the next three numbers will be. Remember that your numbers need to get smaller as you move from left to right. Answer: 215, 213, 211.

- Go through the same steps with more examples and then ask learners to try the example out on their whiteboards or in their maths books, e.g.

  52, 54, 56, 58, _____, _____, _____
  Answer: 60, 62, 64

  2, 4, 8, 16, _____, _____, _____
  Answer: 32, 64, 128

Fives patterns

- Learners can indicate the number pattern on the number chart to discover the number sequence 345, 350, 355, 360, 365.
- Write the following numbers on the board

  _____, _____, 545, 550, 555

  - Discuss the number pattern using the following questions:
  - Which pattern do the numbers belong to? 5s pattern.
  - Are the numbers getting bigger or smaller as we move from left to right? Bigger.
  - Are the numbers that are missing on the left or right? On the left.
  - Are the missing numbers going to be bigger or smaller than 145? Smaller, because they are on the left of 145.

- Go through the same steps with more examples and then ask learners to try them out on their whiteboards or I their maths books, e.g.

  754, 764, 774, 784, _____, _____, _____
  Answer: 794, 804, 814

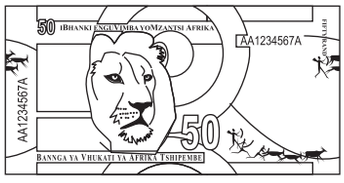
  95, 90, 85, 80, _____, _____,_____
  Answer: 75, 70, 65

- Use the 5s pattern to work out what the first number before 155 will be.

  155 – 5 = 150. Write 150 down on the left of 155.

			155	160	165

Number patterns using money and time.



- Write one set of numbers on the board at a time, making sure that the numbers are not always in order.
- Ask learners "Which one does not belong? Why do you say so?"

  100c, 50c, 250c, 45c, 150c, 200c, 300c

  Answer: 45c does not belong to the 50c pattern

  R47.00, R638.00, R56.96, R572.00, R87.00

  Answer: R56.96 because it is the only amount that has both rands and cents

  R8.25, R4.22, R8.45, R80.00, R21.93

  Answer: R80.00 because it is the only amount that does not have cents

  75c, 175c, 25c, 35c, 50c, 100c, 125c

  Answer: 35c because it is the only amount that does not belong to the 25c pattern

  R50.00, R250.00, R200.00, R100.00, R150.00, R258.00

  Answer: R258.00 because it is the only amount that does not belong to the R50 family

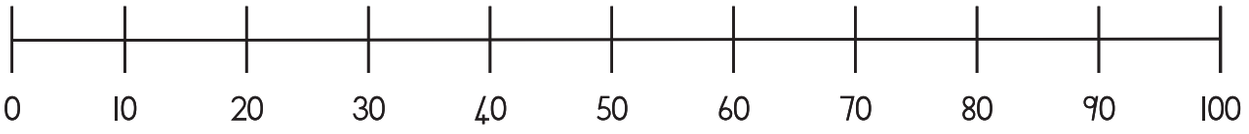
- Write the following numbers on the board:

  _____, _____, R200, R150, R100

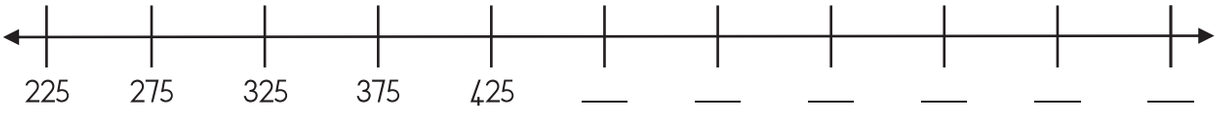
- Ask learners
  - Which pattern do the numbers belong to? R50s pattern
  - Are the numbers getting bigger or smaller as we move from left to right? Smaller
  - Use the R50s pattern to work out what the first number before R200 will be. R200 + R50 = R250
  - Write R250 down on the left of R200.
  - What do we do to find the number before R250? Add R50 again
  - To which number? R250
  - What is the answer? R300
  - Write the complete answer down: R300, R250, R200, R150 and R100

Number patterns and number lines

- Number lines can also be used to teach number patterns to learners (see item 10).
- The number line shows the number pattern in 10s.



Ask learners to complete the number sequence on the number line:



Answer: 475, 525, 575, 625, 675, 725

Notes:

Printable: Numeric patterns (1)

1. Write down the next three numbers in the pattern

a) 104, 109, 114, _____, _____, _____

b) 121, 119, 117, _____, _____, _____

c) 1, 2, 4, 8, _____, _____, _____

d) 25, 50, 75, 100, _____, _____, _____

2. What are the first two numbers in the pattern?

a) _____, _____, 245, 250, 255

b) _____, _____, 155, 150, 145

c) _____, _____, 168, 158, 148

d) _____, _____, 348, 358, 368

3. Complete the table

Which number does not belong?	Give reasons for your answers.
16 22 20 24 28 32	
R5.00 R5.50 R6.00 R6.50 R7.00 R7.70	
R3.25 R4.59 R8.02 R6.00 R1.53	
100c 75c 125c 45c 150c 200c 175c	
R25.00 R56.00 R38.15 R217.00 R387.00	

Printable: Numeric patterns (2)

1. What are the next three numbers in the pattern?

a) R6.40 R6.60 R6.80 _____ _____ _____

b) R12.50 R12.00 R11.50 _____ _____ _____

c) R2 R4 R6 _____ _____ _____

2. What are the first two numbers in the pattern?

a) _____ _____ R125 R155 R185

b) _____ _____ 374c 384c 394c

c) _____ _____ R12 R15 R18

d) _____ _____ R120 R150 R180

e) _____ _____ R180 R150 R120

f) _____ _____ R575 R550 R500

3. Find the missing numbers to complete these patterns

659c	669c	679c			
			R59	R57	R55
	40c	50c	60c		

Geometric patterns

ANA 2013 Grade 3 Mathematics Item 8

8. Repeat the diagram pattern once more.

What should a learner know to answer this question correctly?

Learners should be able to:

- Understand the mathematical vocabulary relating to geometric patterns, e.g.: diagram, pattern, copy, repeat, once more;
- Draw shapes or diagrams correctly;
- Identify a specific geometric pattern.

Where is this topic located in the curriculum? Grade 3 Term 1 - 4

Content area: Patterns, Functions and Algebra.

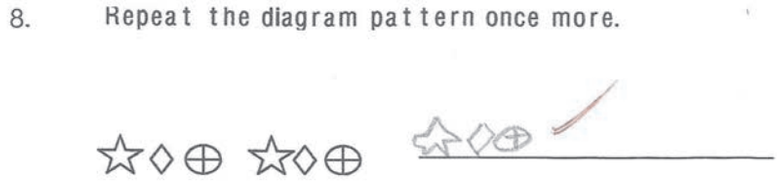
Topic: Geometric patterns.

Concepts and skills:

- Copy, extend and describe in words;
- Simple patterns made with physical objects;
- Simple patterns made with drawings of lines, shapes or objects;
- Create own patterns with physical objects by drawing lines, shapes or objects;
- Patterns all around us: Identify, describe in words and copy geometric patterns in nature, from modern everyday life, from our cultural heritage.

What would show evidence of full understanding?

- If the learner copied the shapes correctly once;
- If the learner was able to draw the shapes and objects correctly.



What would show evidence of partial understanding?

- If the learner copied the pattern correctly but repeated it a second or third time;
- If the learner copied the pattern correctly but then added an additional shape or object to the pattern

8. Repeat the diagram pattern once more.

What would show evidence of no understanding?

- No response to the question;
- If the learner drew a completely different shape or diagram next to the given pattern.

8. Repeat the diagram pattern once more.

What do the item statistics tell us?

88% of learners answered the question correctly.

- Learners did extremely well in this item, showing that the concepts and skills tested in this item, such as repeating geometric patterns, have been mastered by the majority of learners.
- Learners are exposed to geometric patterns from Grade 1 and consolidation of the concepts and skills is evident.

Teaching strategies

Activities to copy, extend and describe patterns made with physical objects, shapes or drawings

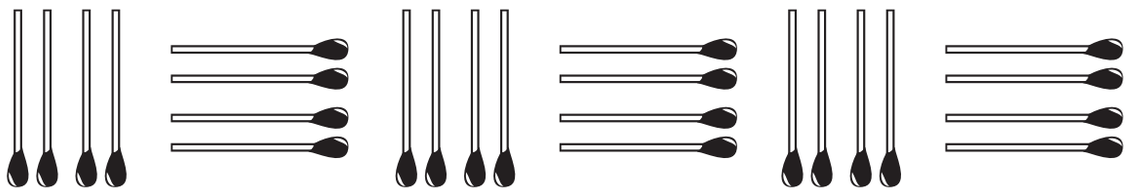
- Patterns can be based on:
  - Natural patterns as found in nature: different petals of flowers, different patterns of leaves;
  - Geometric attributes – shapes: boxes, balls, circles, rectangles, triangles, squares and cylinders;
  - o Relational attributes - sequence: first a red bead, then a blue bead, then repeat the pattern.
- Patterns should be engaged with on a concrete and semi-concrete level before moving to an abstract level to enable learners to understand how to work with patterns.
- This means that you will give learners the opportunity to explore patterns by using physical objects and then later give them drawings to enable them to copy and extend, create and describe patterns.

- Some examples of activities that you could work through with your learners:

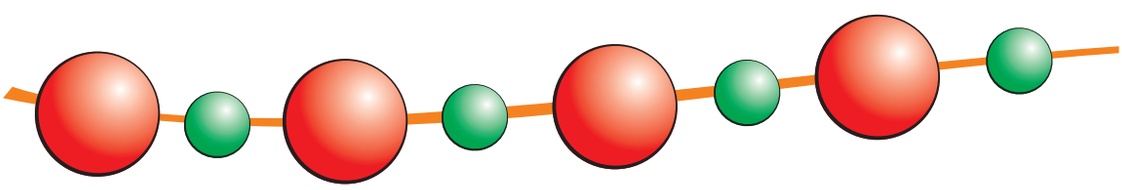
Making patterns:

- Arrange objects in a certain order and repeat the pattern in the same order
- Play games with the learners, e.g. show and look.
  - Learners look at objects which have been placed in a row, e.g. pencil, book, scissors.
  - Cover the objects with a cloth and ask learners to copy the pattern. (They must draw/place the objects in the same sequence).

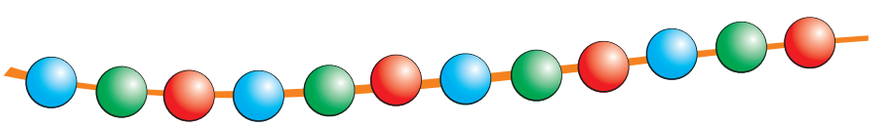
- Learners use matches to build a specific pattern, e.g.



- Threading according to shapes, colours, size.
  - Use beads and a shoelace (threading) to build a pattern. Show learners the pattern to be copied, e.g. red, blue, yellow, green, orange.
  - Use a card with beads drawn on it, e.g. red, blue, green. Learners look at and then copy the pattern.
  - Verbally state the pattern that the learner must thread, e.g. big red bead, small green bead, big red bead.

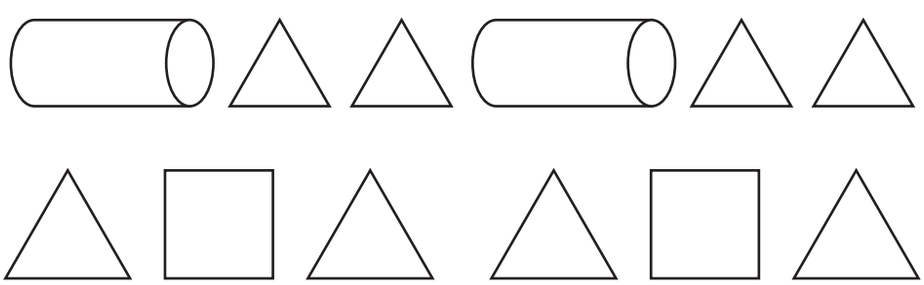


  - Learners can make their own necklaces using clay balls or any objects that can be threaded in a pattern, e.g. bottle tops, macaroni, dry seeds, waste material, etc.



Copying a pattern

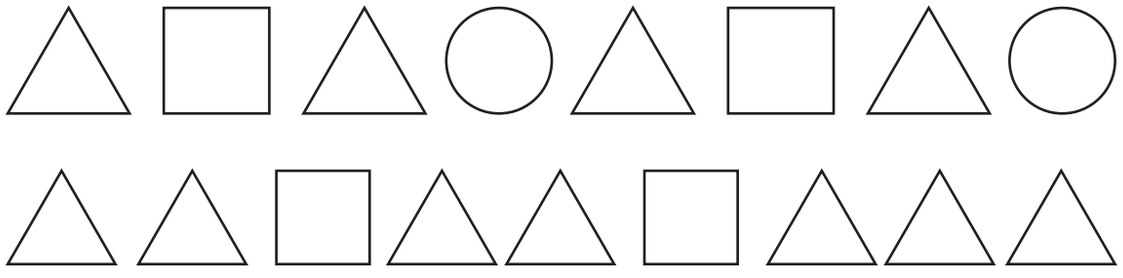
Learners put out shapes or draw shapes to show a pattern, e.g. draw these two patterns on the board:



- Discuss each pattern with your class.
  - The first pattern is made of a cylinder and two triangles. This is then repeated.
  - Draw some repeats of the pattern cycle on the board with the class.
  - The second pattern is made of a square, triangle and square and then repeated.
  - Draw some repeats of the pattern cycle on the board with the class.

Extending a pattern

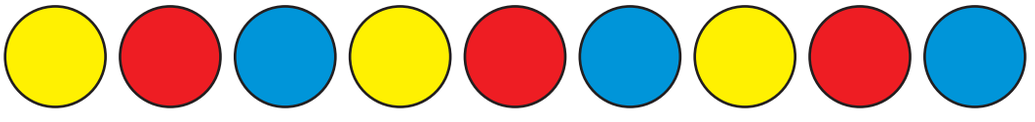
Repeat a pattern two or more times, e.g. draw these two patterns on the board:



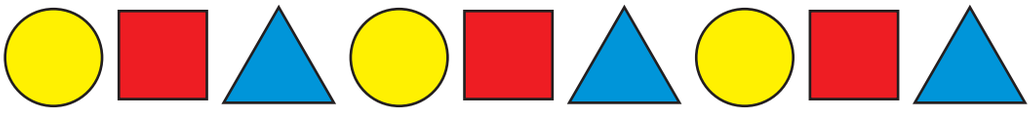
- Discuss each pattern with your class as you did when you worked on copying the pattern.
- Draw some repeats of the pattern cycle on the board with the class.
- A pattern should be repeated at least twice to make it possible for learners to identify the specific pattern.

- Show objects or shapes on the overhead projector.
  - Learners copy the pattern using plastic or paper shapes **(see printables)**.
- Give learners verbal instructions to copy a pattern, e.g. square, square, circle.
  - Learners can use cut-out shapes to build the patterns as demonstrated by the teacher.
  - Learners discuss and copy the observed patterns.

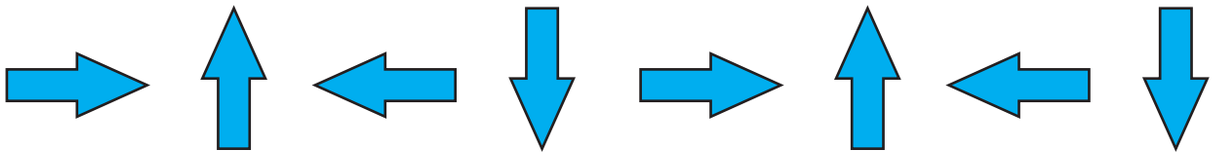
- Patterns can be made by using one shape but changing the colours of the object in a regular way, e.g.



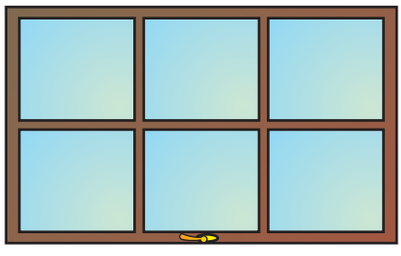
  - Patterns can have different shapes and colours, but they still make a pattern, e.g.



- Some patterns involve rotations, e.g. in which direction will the seventh arrow point? In which direction does the arrow rotate?



- Give learners different kinds of incomplete patterns and ask them to complete and copy the patterns **(see printables)**.
- Patterns around us
- Man-made objects show us examples of patterns. Patterns can be seen in the way bricks, tiles and windows are laid out.
- Discuss the difference between man-made and natural patterns.
- Learners observe, identify and discuss different patterns in the classroom that are repeated to make up a whole, e.g. windows, bricks, wall, curtains, dress of teacher, pencil case, etc.
- Learners should be able to identify that some shape/geometric/natural patterns are based on numbers (quantity), e.g. 6 small window panes make a big window.
- Learners may draw their own pictures and indicate the observed pattern, or the teacher may provide the learners with the outline of an object, e.g. a dress/wall and ask the learners to copy the given patterns within the outline.



- Learners observe/identify any patterns in their environment:
  - Observe patterns in pictures, real objects, traditional clothes and houses.
  - Make patterns on different cultural huts and houses.
  - Go outside and observe patterns in nature or on objects. Guide learners to look at leaves, trees, sand, flowers. Learners could discuss the patterns they have observed in nature. Copy patterns of leaves by etching the leaves on white paper. Learners could observe the patterns on the leaves.
  - Look for natural patterns we make. Almost all natural patterns are symmetrical, e.g. legs on a caterpillar, leaves, elephant footprints.

- Learners in Grade 3 bring objects or pictures of things that are symmetrical and things that are not symmetrical. They should be able to give reasons why the object is or isn't symmetrical.
- Allow learners to talk about the patterns they see in the natural objects that they have shown to you. You could use a worksheet to enable this discussion **(see printables)**.

- Cultural patterns can also be shown to learners. For example, learners can copy Ndebele patterns.

Another example of how geometric patterns can be tested

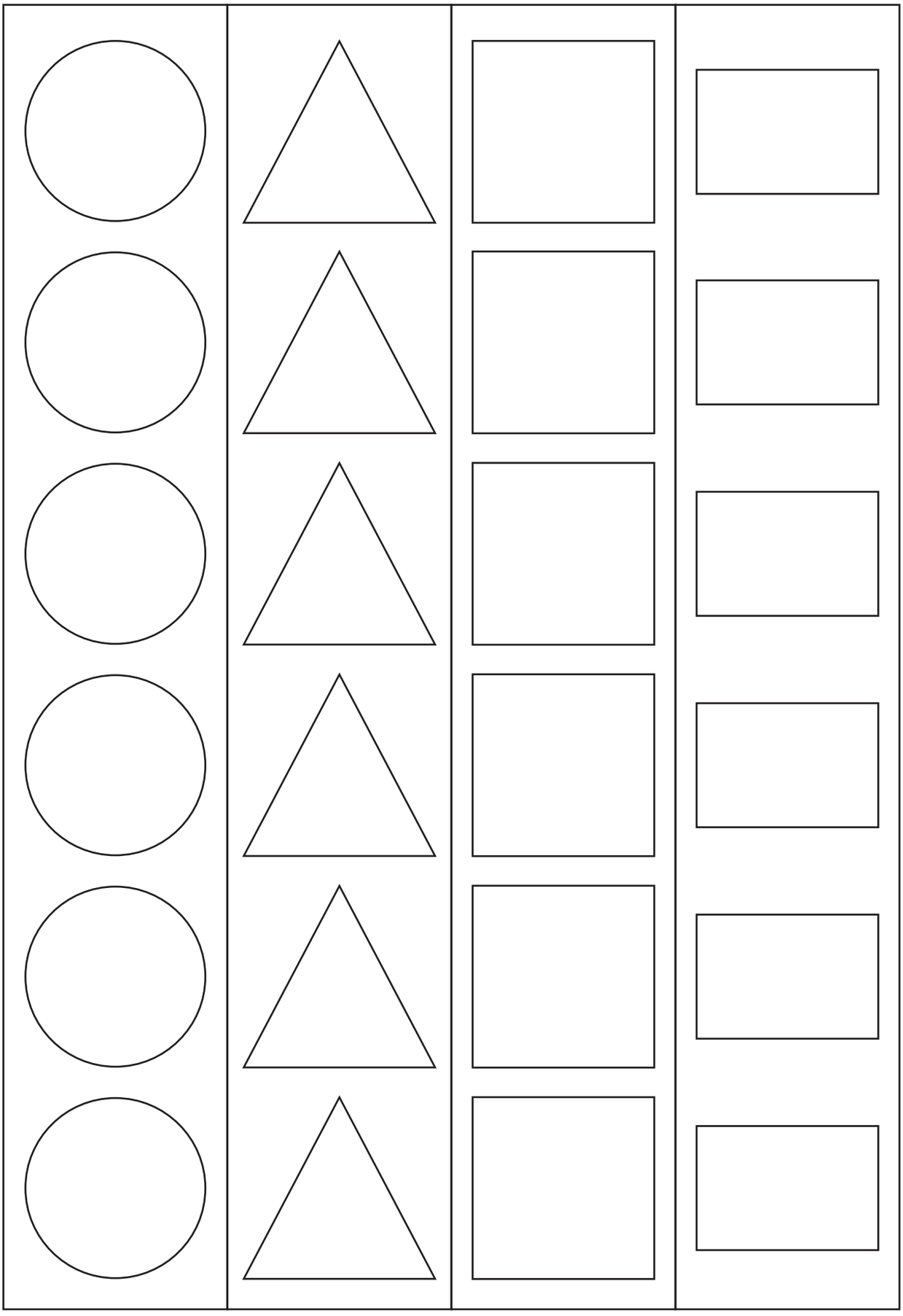
ANA 2014 Grade 3 Mathematics Item 7.1

7.1 Repeat the pattern once.

_____

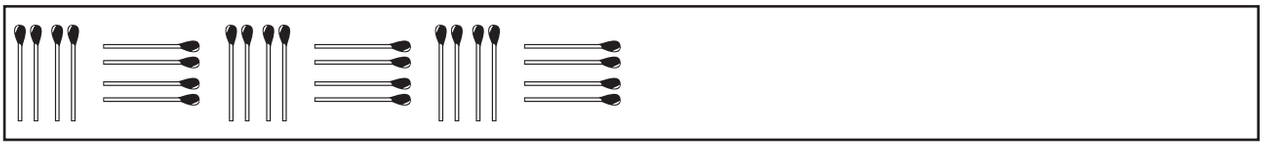
Notes:

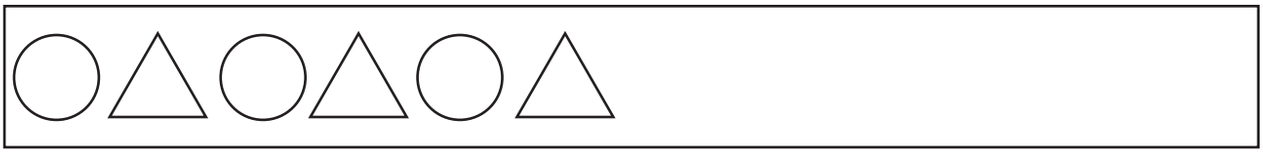
Printable: 2-D shapes



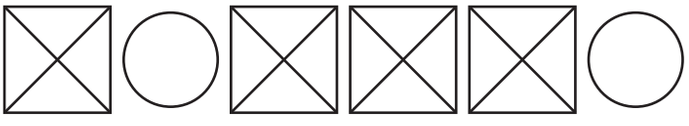
Printable: Geometric patterns

Complete the pattern





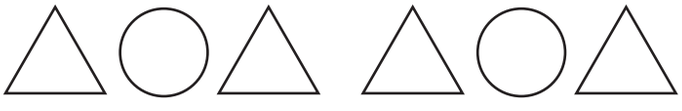
Repeat the pattern once more



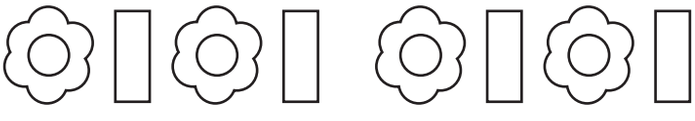
____ ____ ____

Copy the pattern once more

____ ____ ____



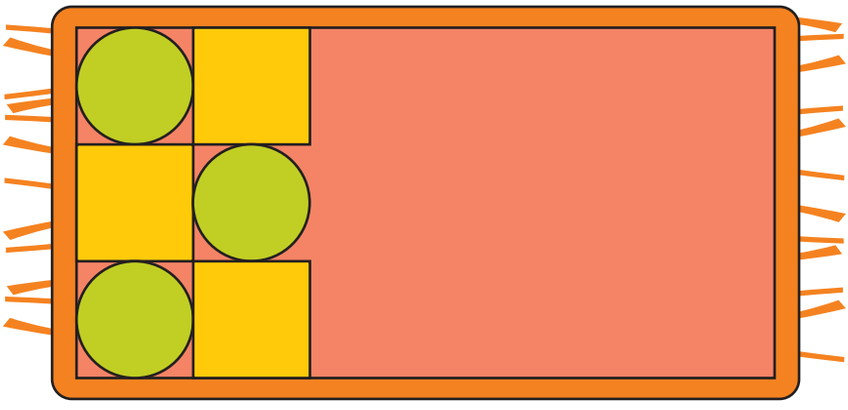
____ ____ ____



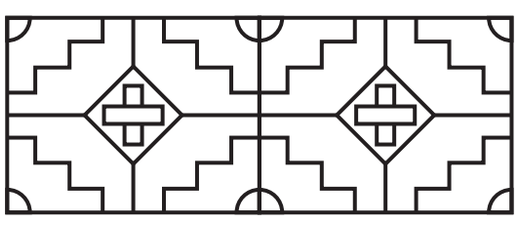
____ ____ ____

Printable: Patterns around us

Complete the pattern on the carpet.



What patterns do you see below?

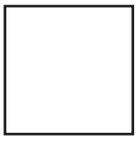


Draw your own pattern

2-D shapes

ANA 2013 Grade 3 Item 7.2

7. Write the name of the given 3-D object and 2-D shape

7.2. 

_____

What should a learner know to answer this question correctly?

Learners need to be able to:

- Read the question and understand the meaning of the term 2-D shape;
- Identify and name 2-D shapes;
- Understand the features/properties of 2-D shapes;
- Know and write the mathematical vocabulary correctly in their home language: square.

Where is this topic located in the curriculum? Grade 3 Terms 1-3

Content area: Space and shape.

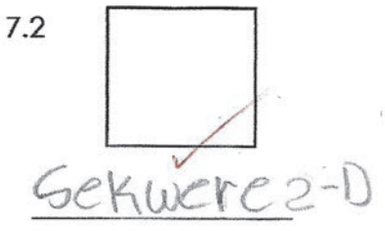
Topic: 2- D Shapes.

Concepts and skills:

- Name and group shapes.

What would show evidence of full understanding?

- The learner shows full understanding by writing the 2-D shape's name correctly.

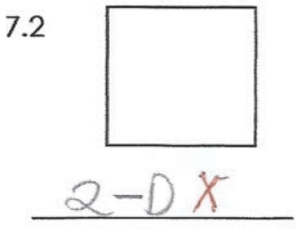


What would show evidence of partial understanding?

- If the learner wrote the name of the 2-D shape, but spelled it incorrectly;
- If the learner wrote rectangle instead of square: this demonstrates that the learner does not

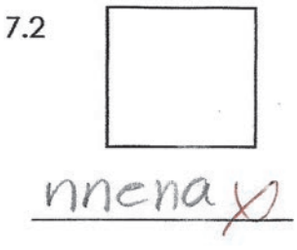
understand the features of a square, e.g. that a square has 4 corners and has 4 sides of equal length;

- If the learner re-wrote part of the question, e.g. 2-D or 2-D shape.



What would show evidence of no understanding?

- If the learner gave an answer that was completely wrong, e.g. wrote the incorrect name for the 2-D shape;
- If no response was given to the question.



What do the item statistics tell us?

61% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- The question was fairly well answered by learners, but the learners need support to spell the names of 2-D shapes correctly.
- Although the 2-D shape "square" is covered in the Grade 1 curriculum content, learners confuse a rectangle and a square. This demonstrates that learners do not understand the features/properties of a square, e.g. a square has 4 corners and has 4 sides of equal length.

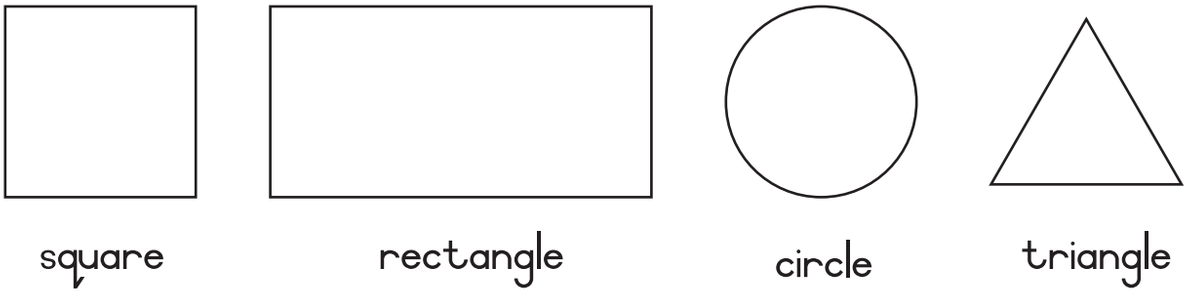
Teaching strategies

Identifying, naming and describing 2-D shapes

- Learners should be able to name and describe common 2-D shapes such as circles, squares, rectangles and triangles. They should also be able to identify straight and round sides of shapes and corners of shapes.
- It is important that learners practically engage in working with shapes.
- The correct mathematical language in the language of learning and teaching (LoLT) of the phase

should at all times be used to consolidate the mathematical terminology.

- Collect a variety of transparent coloured plastic 2-D shapes, such as rectangles, squares, triangles and circles to carry out the following activity in your class:
  - Display the 2-D shapes on your chalkboard or using your overhead projector;
  - Model the pronunciation of the shape names while showing flashcards of the names;
  - Ask learners to match the name cards to the shapes which are displayed;
  - Move the labels to the bottom of the chalkboard;
  - Ask the learners to work in pairs and to name each shape and match the labels to the shapes by pointing.

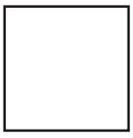
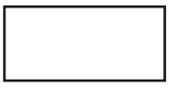
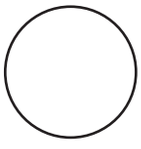
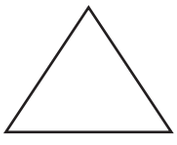


- Remove one shape at a time while learners have their eyes closed.
  - The learners have to draw the missing shape and copy its label from the board.
  - Learners should spell the word correctly.

- Learners are each given a 2-D shape and are asked to find other students who have shapes with similar properties.
  - Learners then explain what the common properties/features are for each group of shapes, for example:

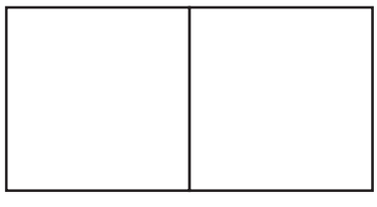
    We all have shapes with three sides.
    We all have shapes with curved edges/sides.
    We all have shapes with four sides that are the same length.
    We all have shapes with two short sides and 2 long sides.

- Ask learners to record the features of the 2-D shapes on the features chart **(see printables)**.

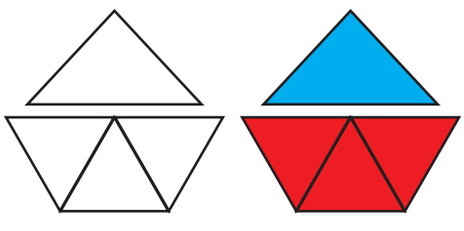
Draw the 2-D Shape	Name the 2-D shape	Number of corners	Number of straight sides	Number of round sides
	square	4	4	0
	rectangle	4	4	0
	circle	0	0	1
	triangle	3	3	0

Working with 2-D shapes to make other shapes

- Use printable 2-D shapes **(see printables)** to make other shapes by joining them.
  - Model the mathematical language while the learners discuss the features of the shapes they are constructing, e.g. Mpho has made a shape with two squares; it has two long sides and two short sides. It is a rectangle.



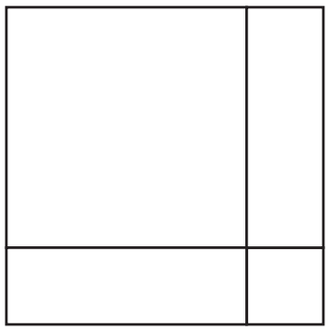
  - Learners display their pictures and describe the shapes they used to make them.

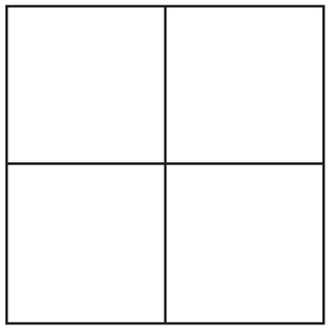


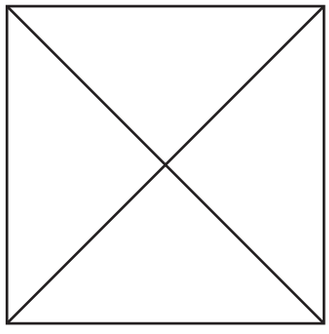
Counting shapes

- Ask learners to find the number of rectangles, triangles and squares in the diagrams below and to explain how they know they have found all of the rectangles, triangles and squares.
- Learners write the names of the shapes and number of shapes counted **(see printables)**.

Count the number of rectangles, triangles and squares in each diagram:



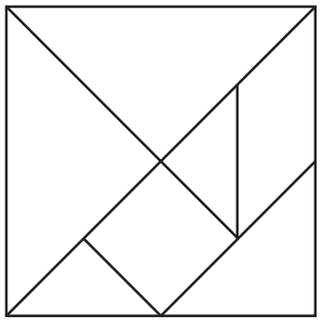


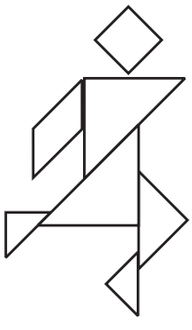


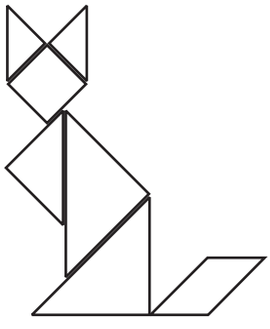
Making patterns using 2-D shapes

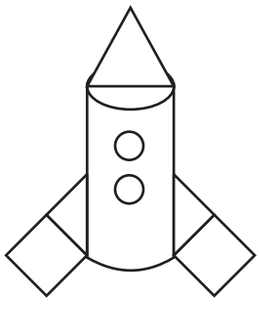
- Learners can spend time working with 2-D shapes in pattern activities. This will help them to consolidate their understanding of the shapes, the names of the shapes and their characteristics. It will also consolidate learners' understanding and knowledge of patterns.

- Ask learners to create pictures with patterns. Learners can use shapes or tangram pieces to construct pictures **(see printables)**.

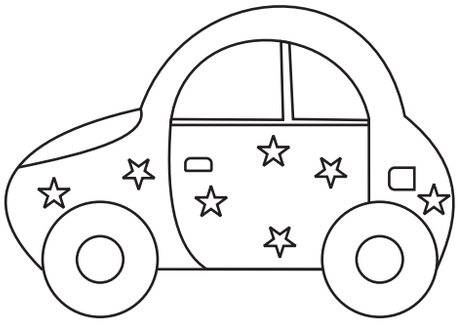


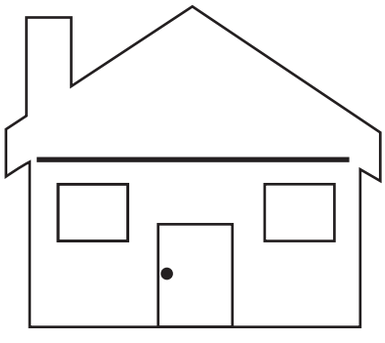






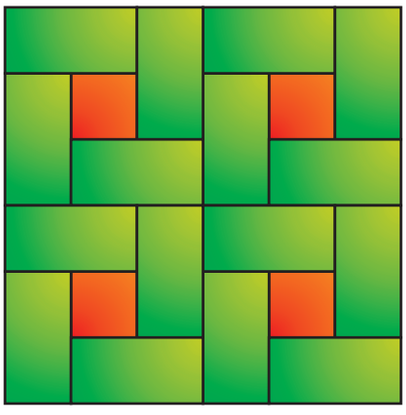
- Colour in a picture with a pattern, e.g. fill the picture of a car with stars or the house with patterns **(see printables)**.





- Fill in a shape with a specific pattern, e.g. use the shapes from the Grade 3 curriculum (circle, square, rectangle, triangle).

- Learners should observe the patterns of floor or wall tiles and draw them, e.g.

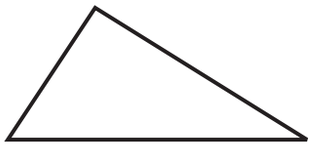


- Learners can decorate the covers of their books with patterns and decorate invitations or cards such as mother's and father's day cards, etc.

Another example of how knowledge of shapes can be tested

ANA 2014 Grade 3 Item 9

9. Write down the name of the given shape below.



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Notes:

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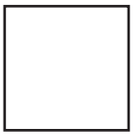
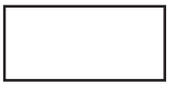
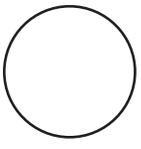
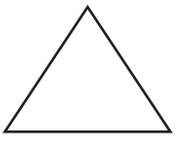
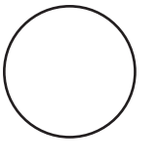
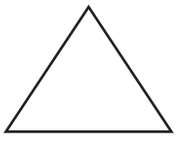
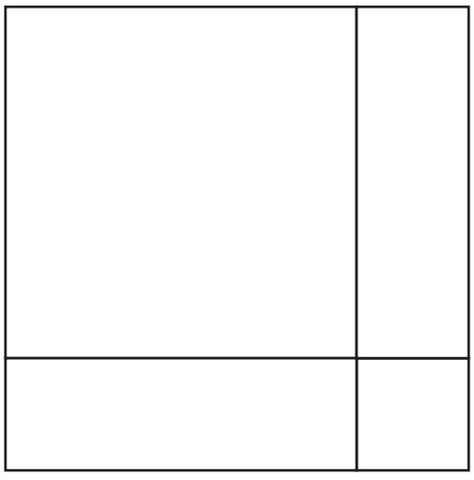
Printable: 2-D shape features chart

Draw the 2-D Shape	Name the 2-D shape	Number of corners	Number of straight sides	Number of round sides
				
				
				
				

Draw the 2-D Shape	Name the 2-D shape	Number of corners	Number of straight sides	Number of round sides
				
				
				
				

Printable: Counting shapes

Count the number of rectangles:



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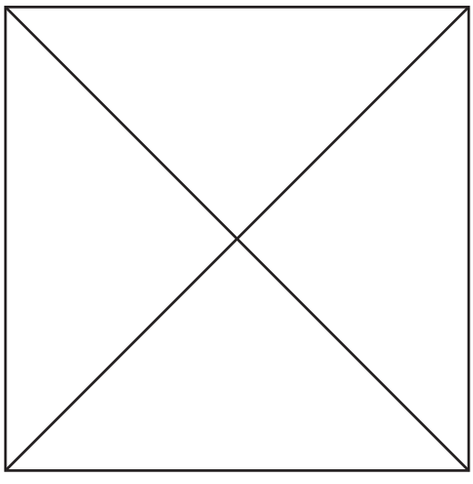
Count the number of triangles:



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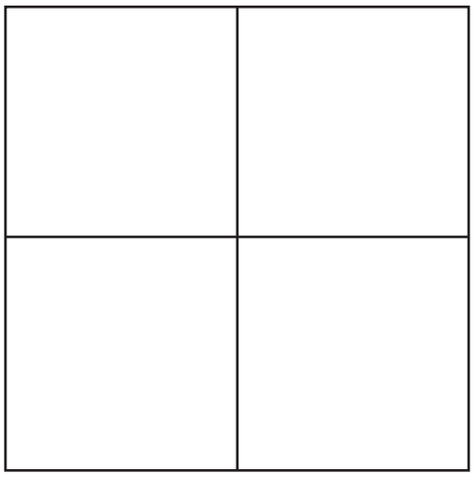
Count the number of squares:



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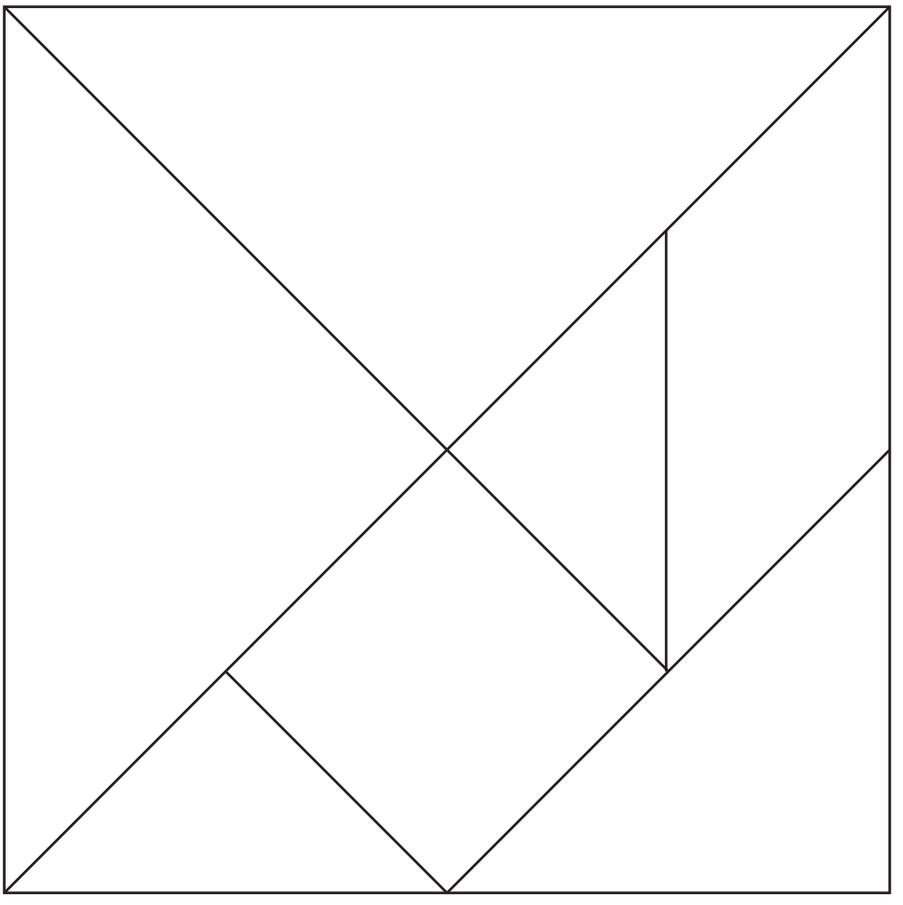
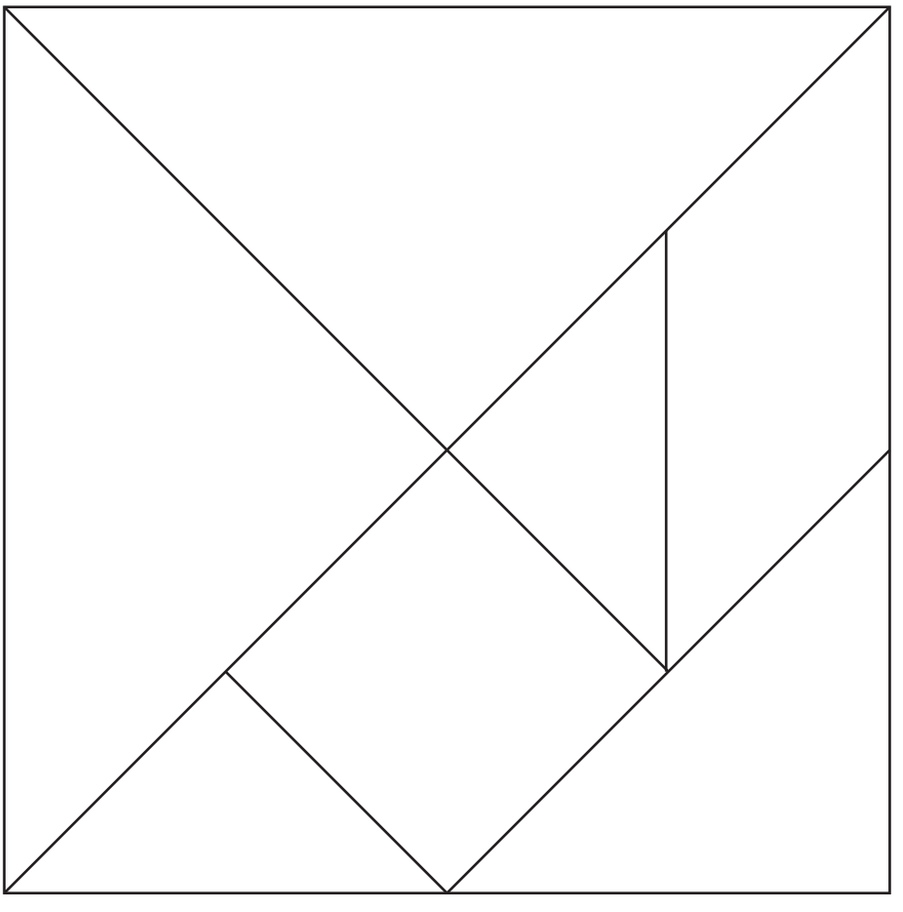
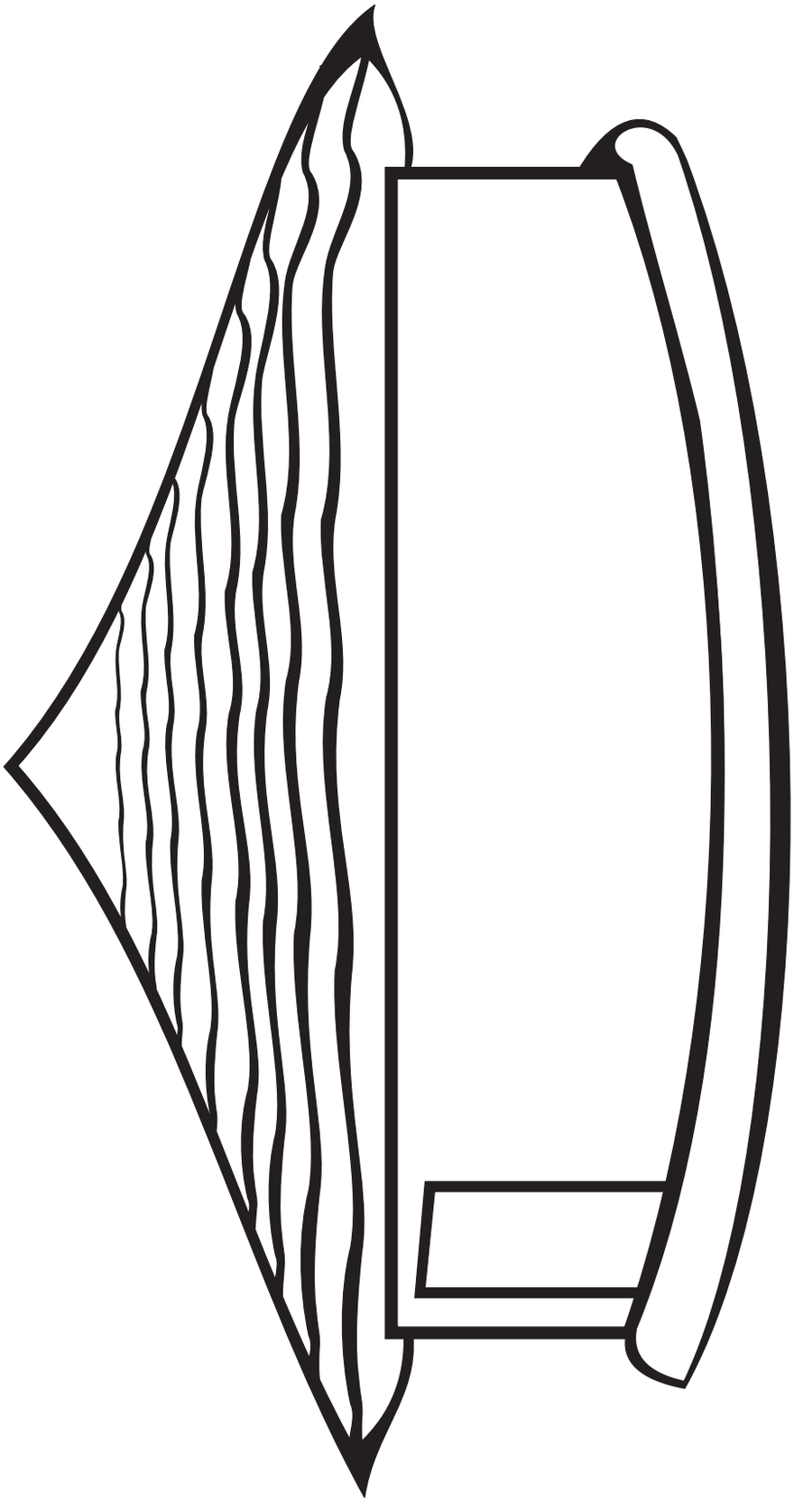
Printable: Tangram





Printable: Colouring patterns in real life contexts

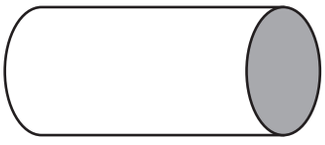
Draw a pattern on your house using 2-D shapes like circles, squares, rectangles or triangles.



3-D objects

ANA 2013 Grade 3 Item 7.1

7. Write the name of the given 3-D object and 2-D shape

7.1.  _____

What should a learner know to answer this question correctly?

Learners need to be able to:

- Read the question and understand the meaning of the term 3-D object;
- Identify and name 3-D objects;
- Understand the features/properties of 3-D objects;
- Know and write the mathematical vocabulary correctly in their Home Language, e.g. cylinder.

Where is this topic located in the curriculum? Grade 3 Term 2

Content Area: Space and Shape.

Topic: 3.2. 3-D Objects.

Concepts and skills:

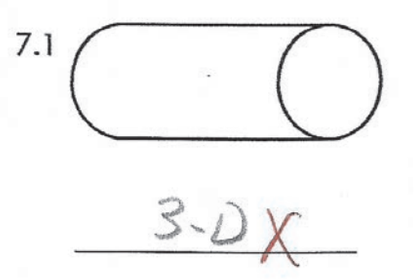
- Recognise and name 3-D objects in the classroom and in pictures: ball (sphere), box (prism), cylinder, pyramid and cone.

What would show evidence of full understanding?

- Full understanding is shown if the learner wrote the 3-D object's name correctly.

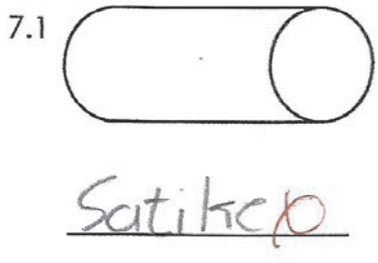
What would show evidence of partial understanding?

- If the learner wrote the name of the 3-D object, but spelled it incorrectly;
- If the learner wrote 3-D or 3-D shape instead of the name of the 3-D object;
- If the learner wrote the name of any 3-D object this demonstrates that the learner realised that the shape is a 3-D object, but does not understand the features of a particular 3-D object, e.g. that a cylinder has 2 faces that are circles, its side is round and it can roll.



What would show evidence of no understanding?

- If the learner gave a completely wrong answer by writing an incorrect, unrelated name for the 3-D object;
- If no response was given to the question.



What do the item statistics tell us?

60% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- The question was fairly well answered by learners, but language might have contributed to the difficulty - the learners need support to spell names of 3-D objects correctly.
- Learners need to understand the features/properties of a 3-D object e.g. a cylinder has 2 faces that consist of 2 circles and a round side and it can roll.

Teaching strategies

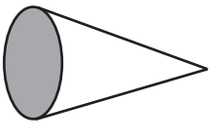
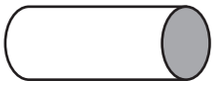
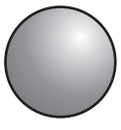
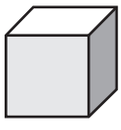
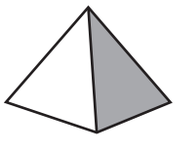
Identifying and labelling 3-D objects

Learners should:

- Recognise and name 3-D objects in the classroom and in pictures: ball (sphere), box (prism),

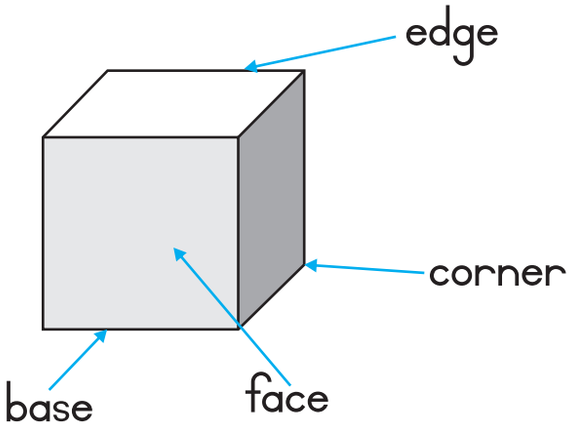
cylinder, pyramid and cone.

- Learners will benefit from practical activities when working with 3-D objects.
- Provide learners with a variety of common 3-D objects, including cones, cubes, cylinders and spheres for them to observe and manipulate.
- Learners should look at the objects from different views.
- Each time learners should name the shape that they are looking at.
- The teacher models the pronunciation of the name of each object while showing flashcards of the names.
- Make name cards for each of the 3-D objects that you work with.
- Together with the learners, match the name cards to the objects which are displayed.
- Make sure that you and the learners always use the correct mathematical names when you refer to and talk about 3-D objects.

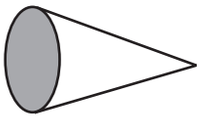
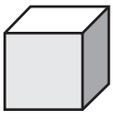
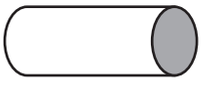
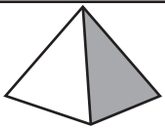
				
cone	cylinder	sphere	cube	pyramid

Features/properties of 3-D objects

- Discuss with learners the features of common 3-D objects (the shape of the faces, the number of corners and edges).

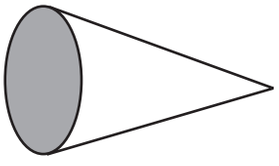
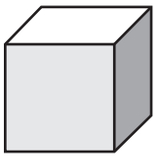
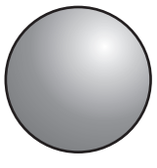
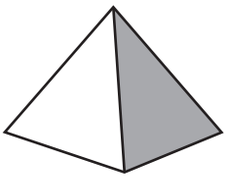


- Use your discussion to help learners complete a chart on the features of each 3-D object **(see printables)**.

3-D objects	Name of 3-D object	Number of faces	Number of corners	Number of edges
	cone	2	1	1
	sphere	1	0	0
	cube	6	8	12
	cylinder	3	0	2
	pyramid	5	5	8

- Ask learners to collect a variety of 3-D objects from around the house or classroom. Ask learners to complete the worksheet **(see printables)** using their collected objects.
- Ask learners to sort their collected objects into groups of the same shape. Ask learners to verbalise their sorting in a report back to the class. Ensure learners are able to verbalise the names of their 3-D objects and to describe their features.

Notes:

3 D objects	Write the name	Write the name of the object or draw the object you brought from home that matches this shape
	cylinder	Glass or 
	cone	e.g. ice cream cone
	cube	e.g. dice
	sphere	e.g. ball
	pyramid	e.g. wooden block

- Learners can also collect a variety of boxes, such as shoe boxes, cereal boxes and match boxes.
- Select a box to display on each learner's desk.
- Ask learners to use a coloured text marker to draw a line over the edges of the box that they can see from their seats.
- Ask learners to select a different coloured text marker to draw a line over the edges they cannot see.

"What am I?" game

- Learners could play the following game to help them learn more about the features/properties of 3-D objects.
- The teacher prepares a set of cards. Each card has a description of a 3-D object.
- For example:

I have 6 faces. They are all the same size and shape. What object am I?

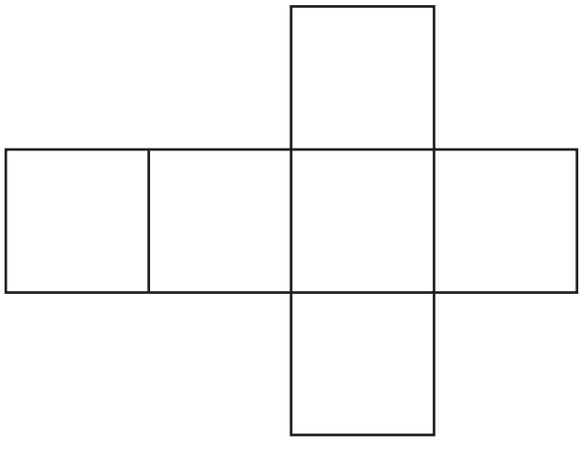
I have 5 faces. 2 are triangles of the same size and 3 are rectangles of the same size. What object am I?

- Arrange the corresponding 3-D objects so all learners can see them
- Ask learners to match the description on the card to a 3-D object.
- Learners write their own 'What am I?' cards to share with a friend.

The relationship between 2-D shapes and 3-D objects

- Learners should use 2-D plastic shapes or printable shapes to explore the relationships between 2-D shapes and 3-D objects.
- Ask learners to visualize the movement of 2-D shapes joined up to make 3-D objects. They should predict the resulting 3-D object.
- For example:
  - I take 6 squares and I join them together to make a 3-D object. What object do I make? (A cube)

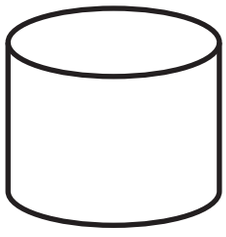
- Learners can cut out printable flat 2-D shapes and build 3-D objects with the cut out shapes using tape to stick them together.



Another example of how to test knowledge of 3D objects

ANA 2014 Grade 3 Item 8

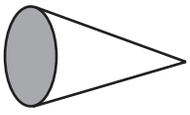
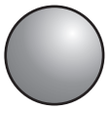
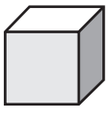
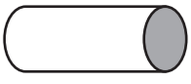
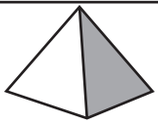
8. Write down the name of the given object below.

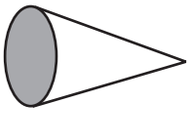
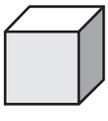
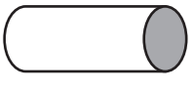
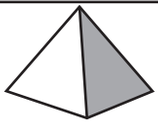


_____

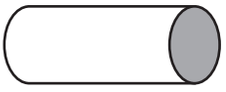
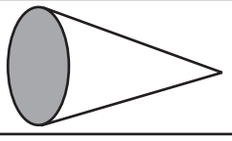
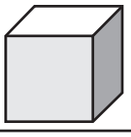
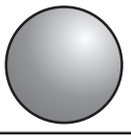
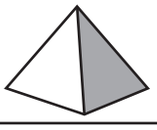
Notes:

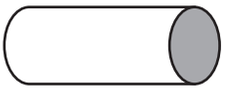
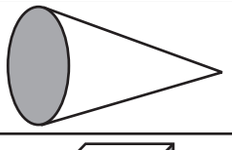
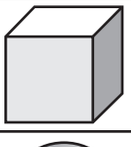
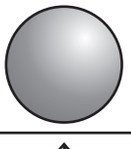
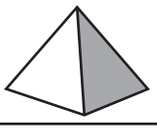
Printable: 3-D features chart

3-D objects	Name of 3-D object	Number of faces	Number of corners	Number of edges
				
				
				
				
				

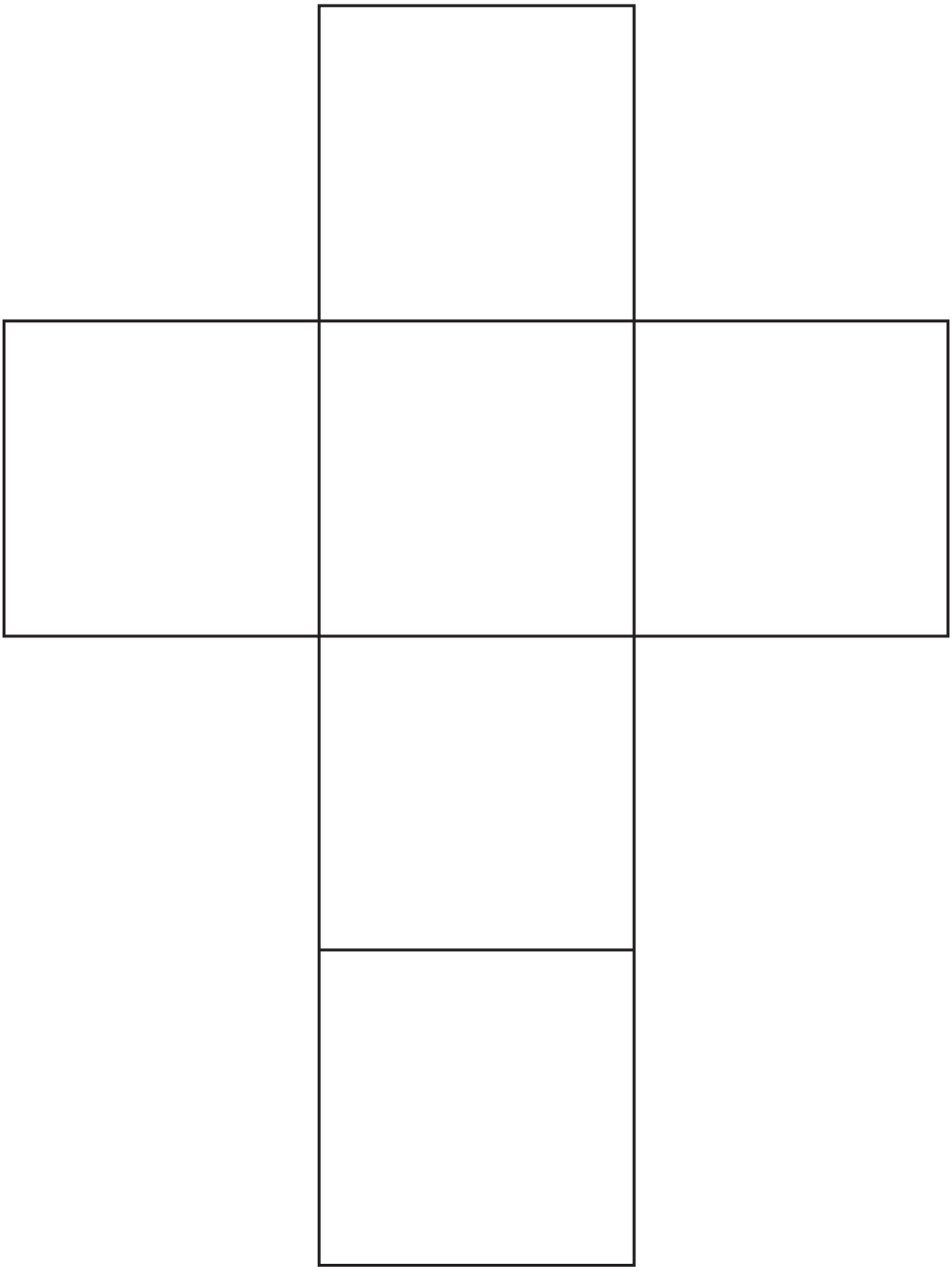
3-D objects	Name of 3-D object	Number of faces	Number of corners	Number of edges
				
				
				
				
				

Printable: 3-D object worksheet

3-D objects	Write the name	Write the name of the object or draw the object you brought from home that matches this shape
		
		
		
		
		

3-D objects	Write the name	Write the name of the object or draw the object you brought from home that matches this shape
		
		
		
		
		

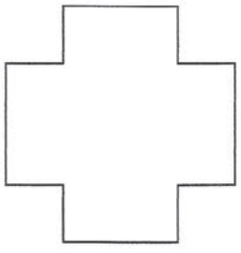
Printable: Cube net



Symmetry

ANA 2013 Grade 3 Mathematics Item 16

16. Draw one line of symmetry on the following shape:



What should a learner know to answer this question correctly?

Learners should be able to:

- Recognise that a line of symmetry divides a shape into two equal parts;
- Recognise that two halves of a shape must be exactly the same as each other.

Where is this topic located in the curriculum? Grade 3 Term 4

Content area: Space and shape.

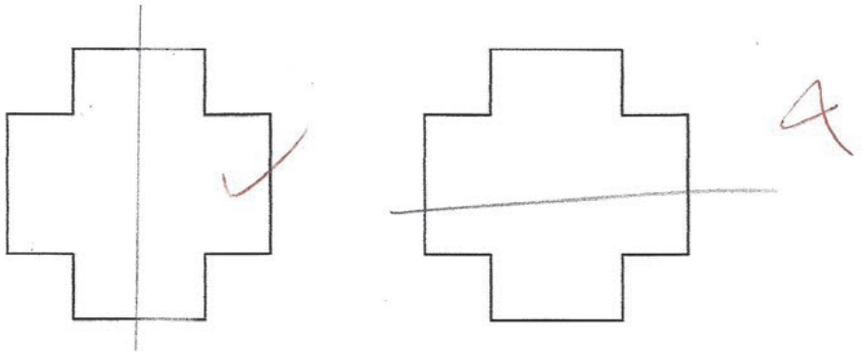
Topic: Symmetry.

Concepts and skills:

- Recognise and draw lines of symmetry in 2-D geometrical and non-geometrical shapes.

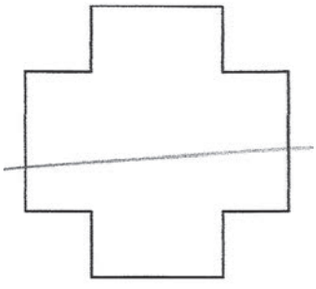
What would show evidence of full understanding?

- If the learner drew one line of symmetry as shown in the following example;
- The learner could have drawn diagonal lines of symmetry, but the line had to be straight and the shape divided into two equal parts.



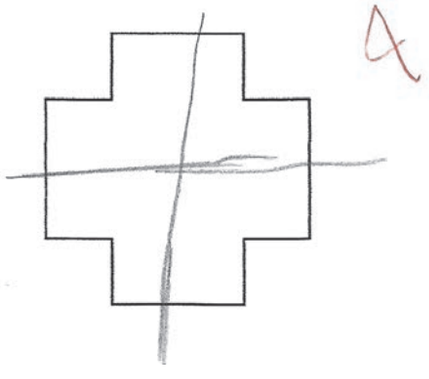
What would show evidence of partial understanding?

- If the learner drew a line dividing the shape into two parts, but the two parts were not equal: this shows the learner recognised that it was necessary to draw a line across the middle of the shape, but did not know that the two halves of the shape had to be exactly the same.



What would show evidence of no understanding?

- If the learner drew lines as shown in the shape that follows:
  - The item asked for one line of symmetry and therefore required an understanding that the shape would then be divided into two halves. If the learner drew the lines as shown in the example, this shows the learner did not understand that only one line was required.
  - The learner did not recognise the error in using two lines and therefore divided the shape into four parts instead of two.
  - In addition to this, the lines drawn do not divide the shape into equal parts, which shows a lack of understanding of the key feature of symmetry.



What do the item statistics tell us?

77% of learners answered the question correctly.

Factors contributing to the difficulty of the item

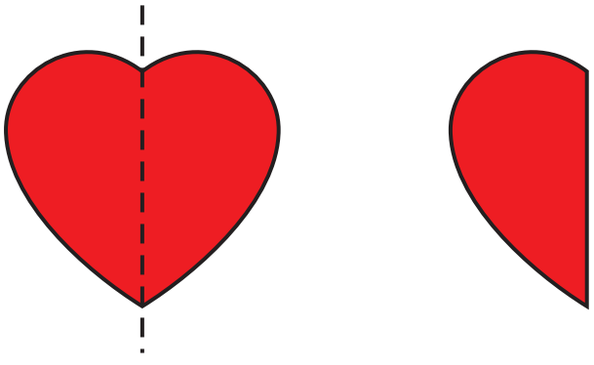
- This item may have been difficult for the learners as identifying lines of symmetry in 2-D shapes is scheduled as Term 4 work.
- Learners may be able to identify lines of symmetry through paper folding, but may struggle to do this in a more abstract way.

Teaching strategies

Paper folding

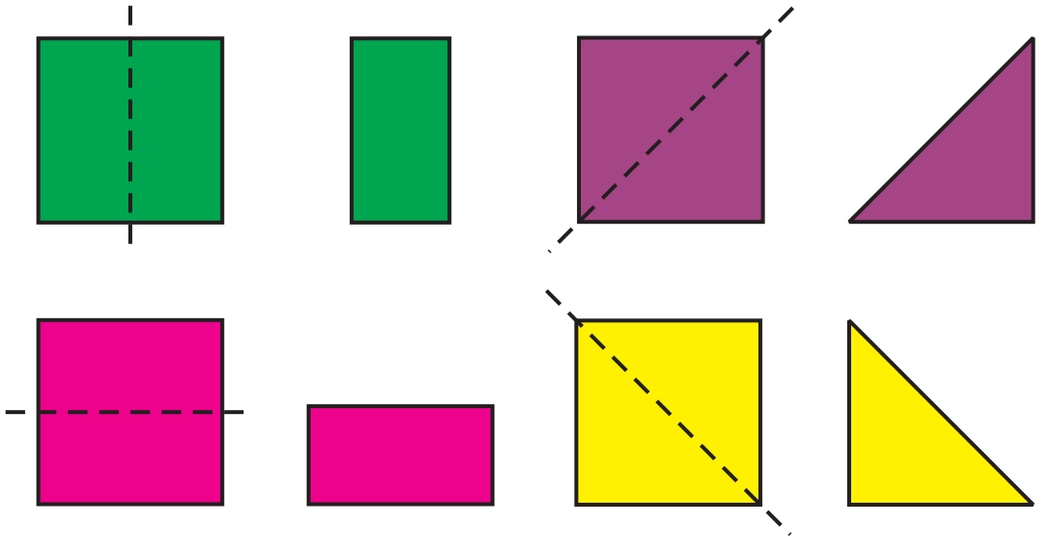
- Ask learners to work in pairs or groups of fours.
- Provide learners with a variety of paper shapes.

- Ask learners to hold up the heart shape.
  - Ask learners to see if they can fold the heart in half so that the one half fits exactly on top of the other half.
  - Ask learners to hold up their folded hearts and say to the learners "Tell me about how you decided to fold your heart shape". Learners may talk about how they folded the heart down the middle so that the two halves could fit exactly together.

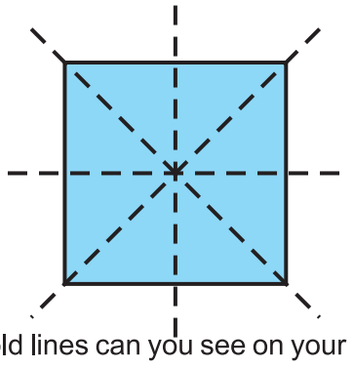


  - Ask learners why they did not fold their heart a different way (for example: the pointed part up towards the bigger part of the heart).
  - Encourage learners to realise that if they had folded the heart a different way they would not have been able to get the two halves to match exactly.
  - Ask learners to open up their heart shapes and to look at the fold line down the middle.
  - Ask learners to use a pencil and ruler to draw a line over the fold line.
  - Explain to learners that the line they have drawn is called the line of symmetry and that it divides a shape into two equal parts.
  - Ask learners "Do you think shapes could have more than one line of symmetry?" Learners should respond "Yes" or "No".

- Ask learners to hold up their square shapes.
  - Ask learners to see if they can fold the square in half so that the one half fits exactly on top of the other half.
  - Ask learners to hold up their folded squares and say “Tell me about how you decided to fold your square shape”. Learners may talk about how they folded the square from left to right, from top to bottom or even diagonally.



  - Ask learners to open up their square shapes and to look at the fold line they have made.
  - Ask learners to use a pencil and ruler to draw a line over the fold line.
  - Ask learners to fold their squares again, but this time in a different way to the first.
  - Ask learners to open up their square shapes and to look at the fold lines they have made.
  - Ask learners to use a pencil and ruler to draw a line over the fold lines.
  - Repeat until the learners have drawn in all the possible lines of symmetry for a square.



  - Ask the learners, “How many fold lines can you see on your square?” Learners should respond, “4”.
  - Encourage learners to realise that when the square is folded on any of the lines of symmetry, that the two halves will be exactly the same.

- Repeat with a variety of shapes.

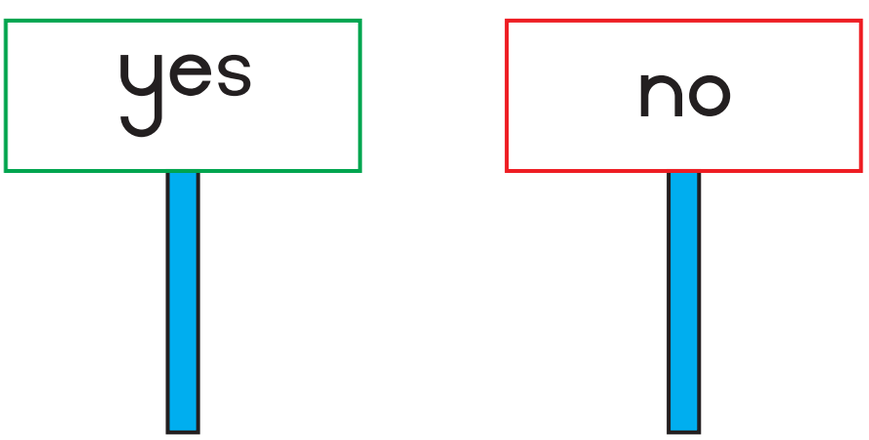
Paper folding and picture matching

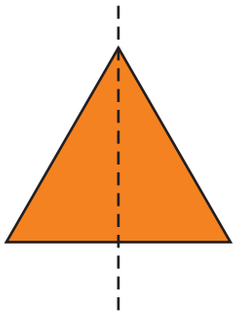
- Ask the learners to work in pairs or groups of four.
- Provide learners with a collection of paper shapes and a set of cut up shape picture cards **(see printables)**. The paper shapes need to match the shapes represented on the shape picture cards.
  - Ask the learners to place the shape pictures face down on the desk and to shuffle them around.
  - Ask the learners to turn over one shape picture card.
  - Ask the learners to then find the corresponding paper shape and to fold it according to the line of symmetry shown on the picture card so that the two folded halves match exactly.
  - Encourage the learners to discuss how they knew that the folded paper shape matches the picture of the shape.
  - The shape picture card can then be removed from the pile and laid to one side.
  - The paper shape may be re-used with another shape picture card which shows a different line of symmetry.
  - The above steps would then be repeated as another shape picture card is selected.

- **NOTE:** The rectangle (last shape on the picture card printable) has a vertical and a horizontal line of symmetry. Allow learners to experiment with folding rectangles to see if the oblique line is a line of symmetry. (It is not. If you fold a rectangle using the opposite corners as the points from which you fold, the two sides that you make do not fit onto each other perfectly. This means the line joining the opposite corners of a rectangle (the diagonal) is not a line of symmetry for the rectangle.)

Symmetry yes/no activity

- This activity can be done with the whole class or with a small group of learners, depending on the resources available to you.
- Make yes/no sticks by pasting the flash cards 'yes' and 'no' back to back at the top of a stick or straw or a tongue depressor. It is a good idea to make the 'yes' card a different colour to the 'no' card.
- Provide each learner with a yes/no stick.





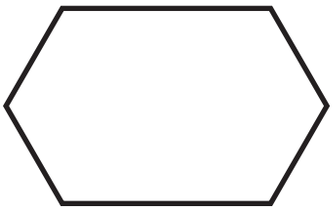
- Draw a shape on the board with a line of symmetry through it.
  - Ask the learners to show you either the 'yes' side or the 'no' side of their sticks, depending on whether they think your drawing has an accurate line of symmetry.
  - Ask the learners “Why did you say that?” and allow the learners an opportunity to verbalise their understanding that a line of symmetry must divide a shape into two equal parts.

- The above steps can then be repeated, with care being taken that not every drawing on the board shows a correct line of symmetry, for example:

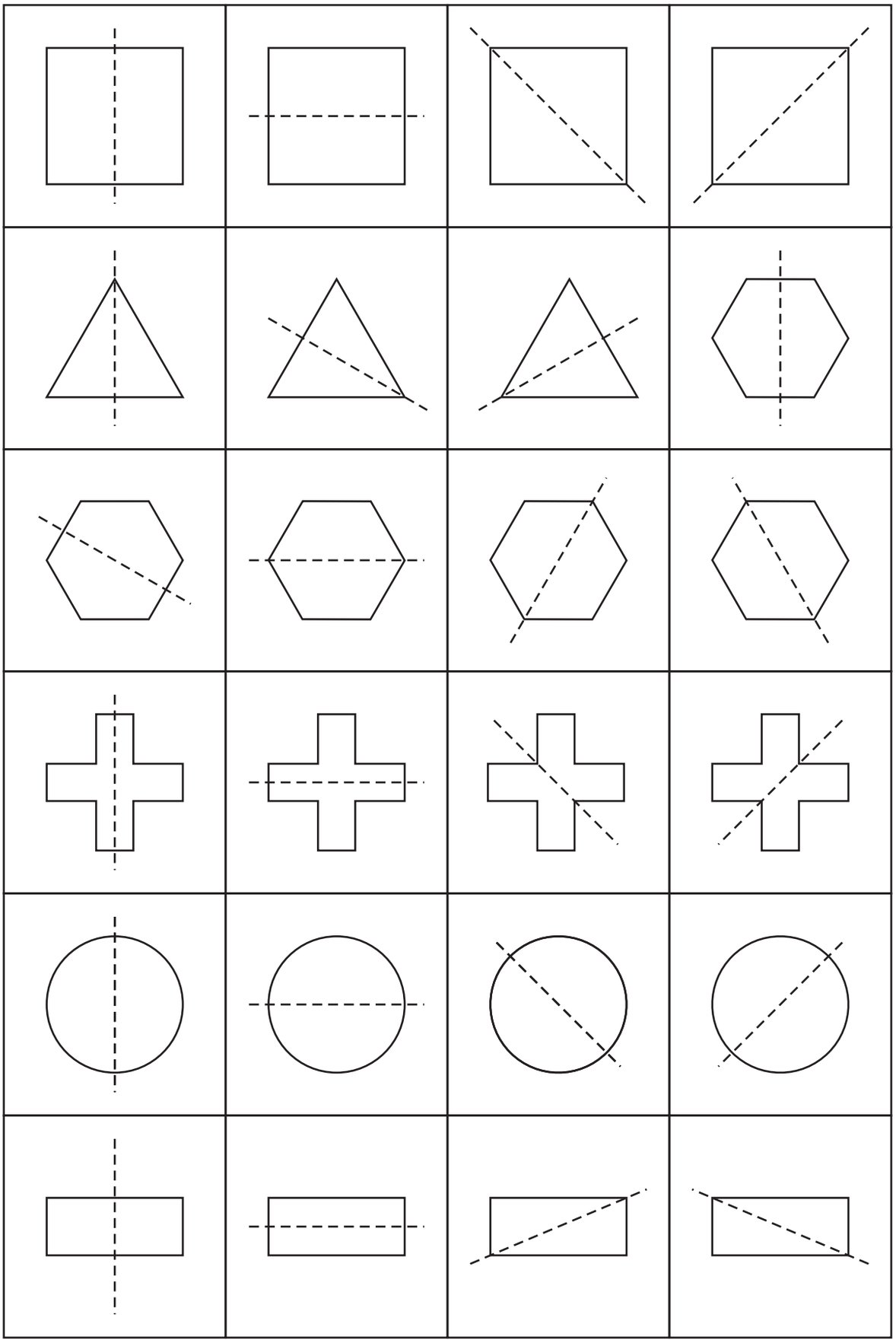
Another example of how symmetry can be tested

ANA 2014 Grade 3 Mathematics Item 10

10. Draw only one line of symmetry on the following shape.



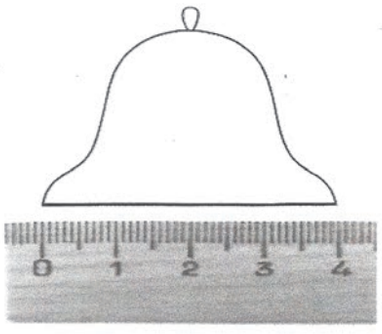
Printable: Shape picture cards



Measurement of length

ANA 2013 Grade 3 Mathematics Item 17

17. Write the width of the bell.



Width = _____ cm

What should a learner know to answer this question correctly?

Learners should:

- Understand the mathematical vocabulary relating to measurement of length, e.g. metre, centimetre, length and width;
- Understand how to read measurement correctly on a ruler;
- Know that length is measured in metres and centimetres;
- Understand the standardised abbreviations for units of measurement, i.e. metre (m) and centimetre (cm).

Where is this topic located in the curriculum? Grade 3 Term 2

Content area: Measurement.

Topic: Length.

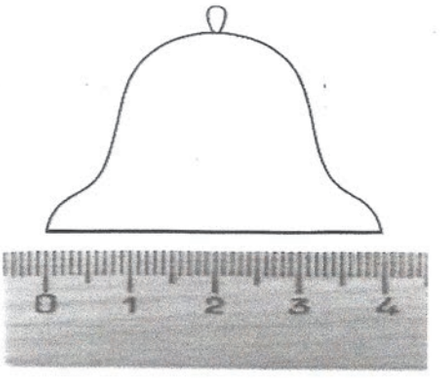
Concepts and skills: Introducing formal measuring;

- Estimate, measure, order and record length using metres as the standard unit of length;
- Estimate and measure lengths in centimetres using a ruler.

What would show evidence of full understanding?

- If the learner answered 4 cm: this shows that the learner is able to read width correctly on a ruler.

17. Write the width of the bell.

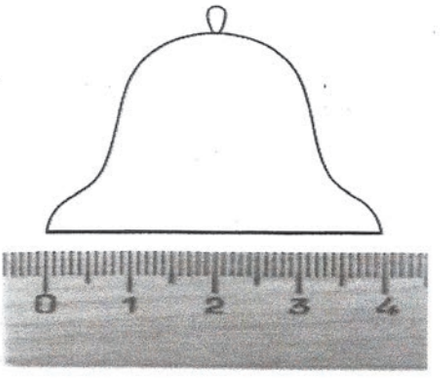


Width = 4 cm

What would show evidence of partial understanding?

- If the learner did not write the correct width of 4 cm but wrote 6 cm: this shows the learner cannot read measurement correctly on a ruler.

17. Write the width of the bell.

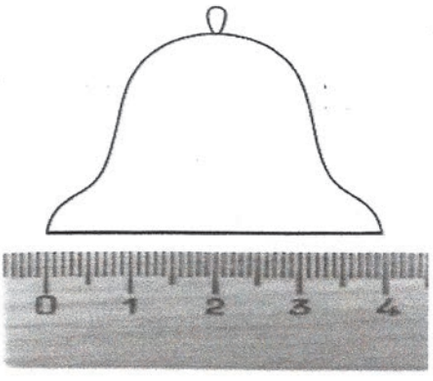


Width = 6 cm

What would show evidence of no understanding?

- No response to the question; or
- If the learner wrote a number that does not even appear on the ruler as the answer.

17. Write the width of the bell.



Width = 20 cm

What do the item statistics tell us?

69% of learners answered the question correctly.

Teaching strategies

- Measurement involves identifying an attribute to be measured (e.g., length, mass, capacity or area) and then using definable, consistent units to find the "how muchness" of the attribute.
- If learners are taught measurement procedures and rules (e.g. formulas) before they understand measurement concepts, they will not fully grasp the meaning of the different attributes, the processes involved in measuring or the significance of the units used to indicate measures.
- It is important for learners to engage in learning situations that help them to understand different measurable attributes and that teach them to measure those attributes in meaningful ways.
- Learners should also be taught all of the terminology relating to the concepts.

Understanding length through non-standardised measurement

- Ask the learners: Did you know that before rulers and tape measures were invented people had to use things that were always with them to measure? They used fingers, arms, hands and feet. People call these measurements arm spans, hand spans (fingers must be stretched out wide), footsteps (the distance between your heel and big toe) and so on.

- Linear measurements are given specific names in particular contexts:
  - ***Length*** is the distance along an object from end to end.
  - ***Width*** is the distance from one side of an object to the other side.

- ***Height*** is the distance from the lowest point to the highest point of an object or a person.
- ***Depth*** is the distance from the top of something to its bottom, from front to back, or from the outside in.
- ***Distance*** is the amount of space between two points.

- Analysing an item in terms of its length:
  - The length of an object can be found by determining the number of units, laid end to end, that make up the distance from one point to another.

- Mathematical vocabulary relating to length:
  - short, tall, long
  - shorter, taller, longer
  - shortest, tallest, longest

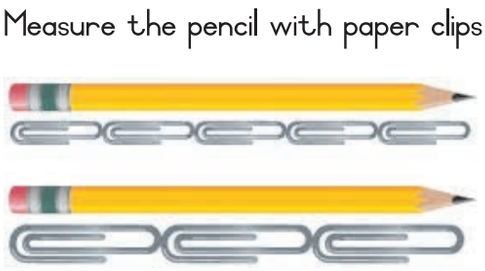
Rules for measuring using non-standardised units

- Measuring length with non-standard units involves counting how many of the chosen units make up the length of the object being measured. For example, the length of a desk could be 8 hand spans (using a small hand as a measure) or 4 hand spans (using an adult hand).
- Choose units of measurement (objects) that are the same size and shape, e.g. paper clips, cubes, pieces of string.
- Learners can measure by putting out a row of the objects being used as units of measurement along the length of the object being measured. The units of measurement must be of the same length and no gaps must be left between them – the objects need to be put out so that they touch each other.
- If learners use hand spans or footsteps as units of measurement they must place one hand or foot next to the other and then move the first hand or foot to the other side of the second hand or foot, counting each time they place a hand or foot along the object.

Activity 1

- Choose a "starting line" on the object being measured to begin the measurement.
- Ask learners to predict or estimate the number of units of measurement that will make up the length of the object.
- Learners measure the object by placing the units of measurement end to end along the object from the starting line so that each unit touches the one before it.
- Learners count and record the number of units it takes to get to the end of the object being measured.
- Give learners the opportunity to measure different objects, e.g. a pencil, an A4 book, a desk, with 3 different units, e.g. a paperclip, a piece of string, a cube.
- Discuss which object is longer or shorter.
- Ask learners to always state the name of the unit of measurement, e.g. the book is 12 paperclips

wide, etc.

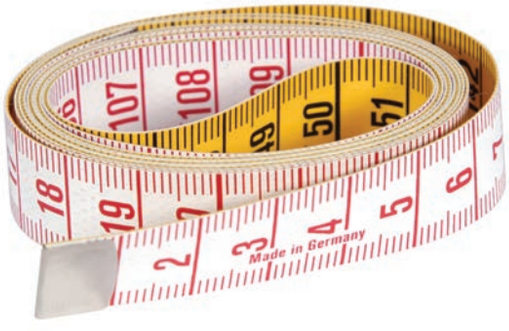


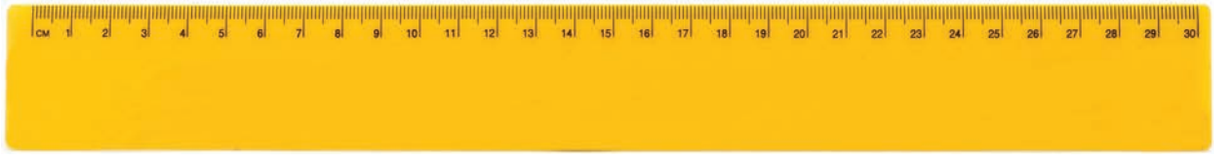
Activity 2

- Divide learners into pairs and ask them to measure their books and desks with hand spans, with straws and with matches.
- Ask learners to put out the straws across the book and then the desk.
- Ask learners to count how many straws they used. Discuss which object is longer or shorter.
- Let each learner measure their books and desks with their hand spans and compare each other's answers.
- Do the same with the matches.
- Ask the learners why they think the answers are not the same. Ask the learners if they think using hand spans is a good way to measure.
- Lead them to the answer that because our hands are not the same size, measurements will differ and so this is not a good way to measure.
- Use the printable sheets **(see printables)** and non-standard units (such as paper clips) to find the lengths of the objects.

Understanding length through standardised measurement

- Learners should be formally introduced to standardised measuring instruments: Metre sticks, tape measures and measuring tapes measure in metres (m), whereas rulers measure in centimetres (cm).





Measuring in metres

- Show learners 4 pieces of wool/string of different lengths.
  - Ask 4 learners to hold the pieces up from end to end so that learners can compare the lengths.
  - Now ask learners to order the pieces from the shortest to the longest and vice versa.
  - Say "We are going to use metres to measure the lengths".
  - Show the learners the metre stick and ask them to compare the 4 pieces of wool/string to the length of the stick.
  - Ask the learners which of the pieces is longer than a metre and which is shorter? (Discuss and consolidate the meaning of the terms longer, shorter and metre).

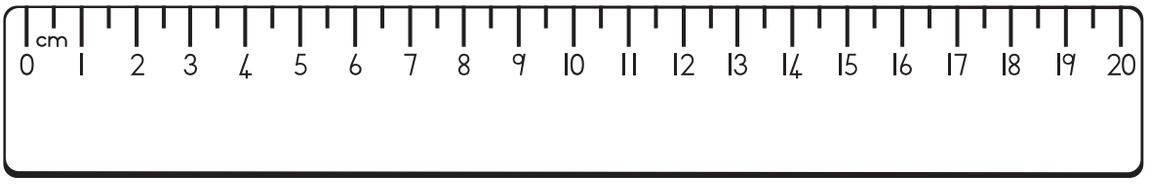
- Ask learners to look around and find what they can see in the class that is shorter/longer than a metre.

- Cut lengths of wool/string 1 m long and show the learners how you measured the lengths with a metre stick.

- Hold up a piece of wool/string that is 1 metre long.
  - Ask learners to estimate how many metres long the mat, shelves, table or any long object in the class is.
  - Ask one of the learners to help you to measure the object by placing the length of wool/string at one end of the object. Mark where the string ends and then flip the string over and reposition it where it ended. Repeat this until you reach the end of the object, counting the number of times the string is placed to give you the object's length in metres.
  - Learners then work in pairs. Give each pair a piece of string that is 1 metre long.
  - Give the learners a list of 3 items and ask them first to estimate how many metres long each item will be. Then ask them to measure the items with the string and record their measurements.
  - Discuss which objects are longer or shorter.

Measuring in centimetres

- After learners have worked with a metre stick, let them work with a ruler.
- Explain to learners how to use a ruler correctly to measure.



  - The zero (0) on the ruler mast be placed at the beginning of the object being measured.
  - The space from 0 to 1 is 1 centimetre and from 1 to 2 is one more centimetre, but altogether the space from 0 to 2 is 2 cm.
  - Use the printable sheets **(see printables)** and a ruler marked in centimetres to find the lengths of

objects.

- Make sure that learners know length is measured in centimetres (cm) or metres (m).

Word problems involving length

- It is important to expose learners to word problems that involve length: Learners must write the number sentence and solve the problem.

- Here are some examples:

a. Lindiwe has made two towers of plastic bricks. The one tower is 24 cm high, and the other is 37 cm high. If she puts one tower on top of the other one, how tall with the new tower be?

b. Thato has a piece of wool that is 129 cm long. If she cuts 115 cm off for a friend, how much will she have left?

c. Bongi has a rope that is 50 m long. Themba has a rope that is 17 m longer than Bongi's rope. How long is Themba's rope?

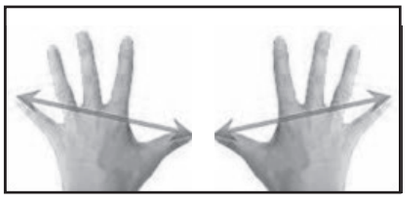
d. Mum has 18 m of material. She wants to share it between her three children. How many metres of material will each child get?

e. Mandy uses ribbon to decorate her gifts. She uses 5 m of ribbon for each gift. She wants to decorate six gifts. How many metres of ribbon must she buy?

Another example of how knowledge of measurement can be tested

ANA 2014 Grade 3 Mathematics Item 14

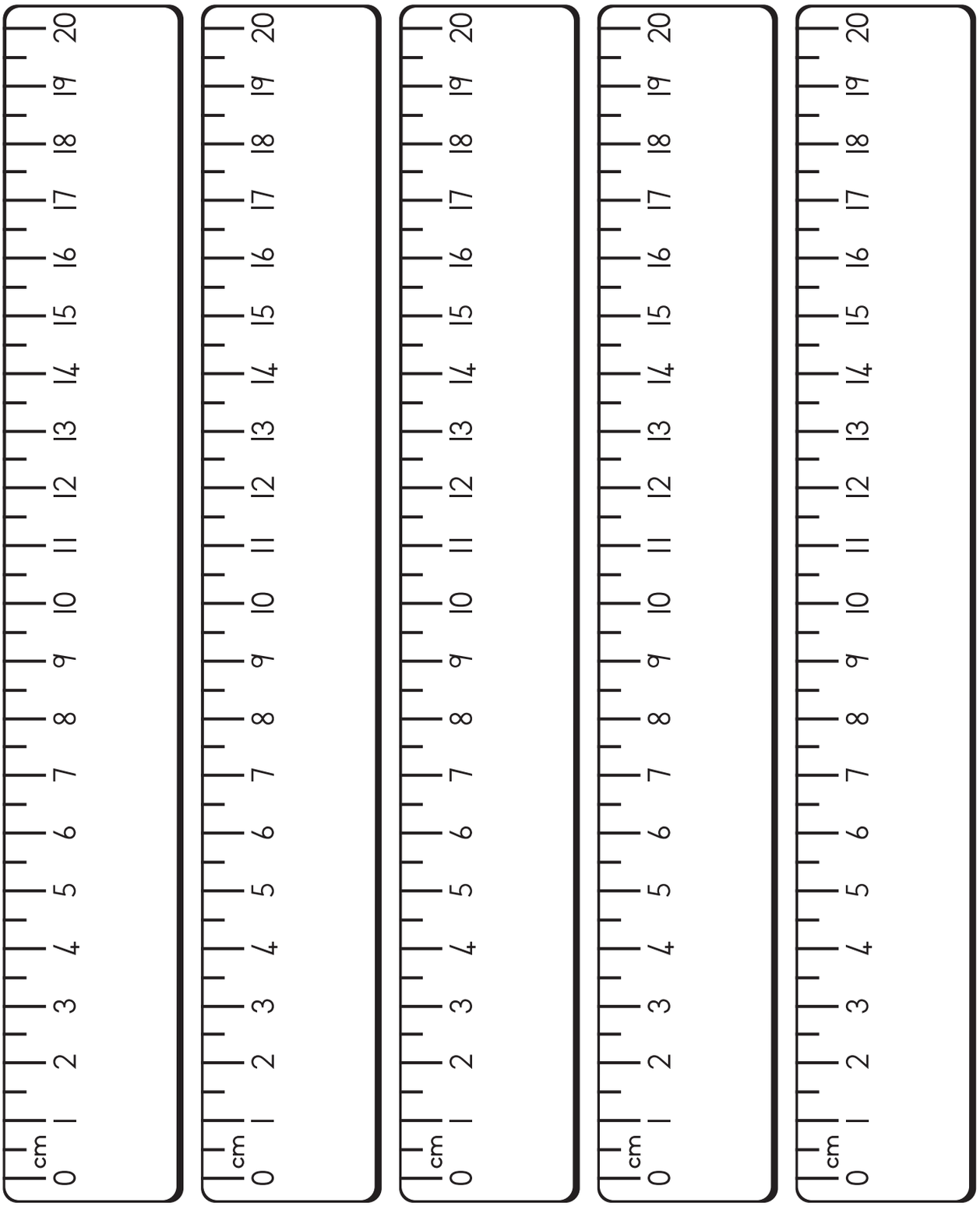
14. The hand span of each hand is 10 cm.



Together the hand spans are _____cm.

Note: This item can also test repeated addition leading to multiplication, i.e. 10 + 10 = 10 x 2 = 20

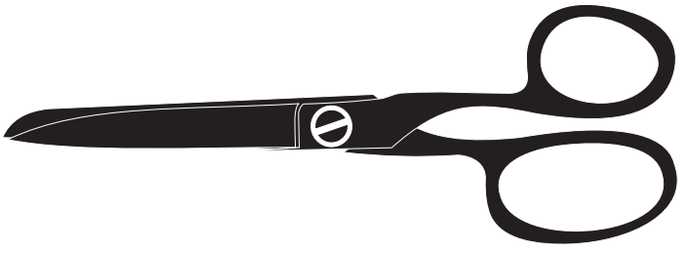
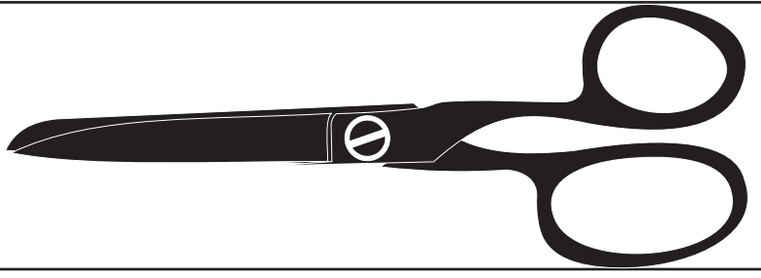
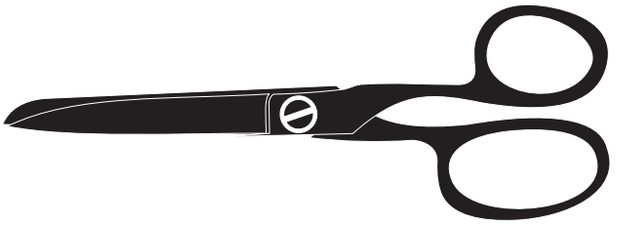
Printable: Centimetre rulers



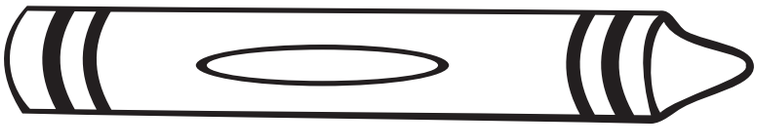
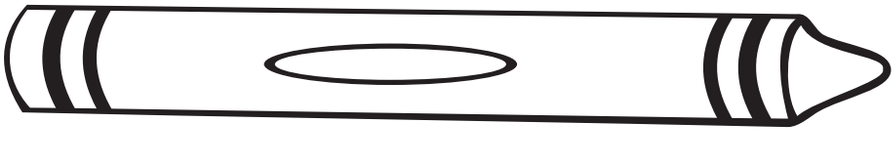
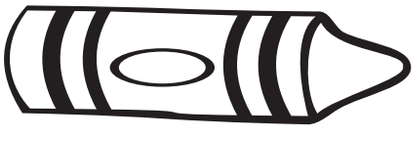
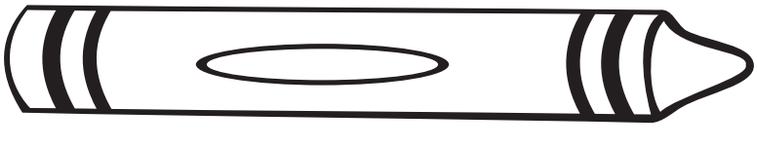
Printable: Length (1)

	Measure the length of the line with your paper clip/ruler	Write the length
1		
2		
3		
4		
5		
6		

Printable: Length (2)

	Measure the length of the scissors with your paper clip/ruler	Write the lenght
1		
2		
3		
4		
5		
6		

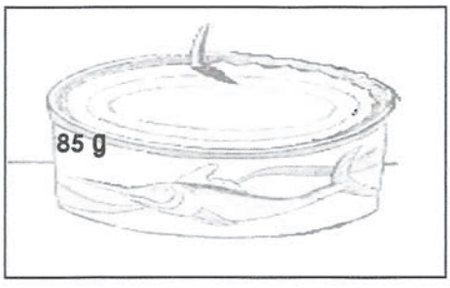
Printable: Length (3)

	Measure the length of the crayons with your paper clip/ruler	Write the length
1		
2		
3		
4		
5		
6		

Measurement of mass

ANA 2013 Grade 3 Mathematics Item 18.1

18. Circle the correct word in brackets for each of the sentences below.



18.1. The mass of the can of fish is measured in (grams, centimetres)

What should a learner know to answer this question correctly?

Learners should:

- Understand the mathematical vocabulary relating to measurement of mass, e.g. mass, measure, kilogram and gram;
- Know that mass is measured in kilograms or grams;
- Understand the standardised unit abbreviations for kilogram (kg) and gram (g) and show understanding of the standardised measuring units;
- Be able to discriminate between measurement of length and mass.

Where is this topic located in the curriculum? Grade 3 Term 1

Content area: Measurement.

Topic: Mass.

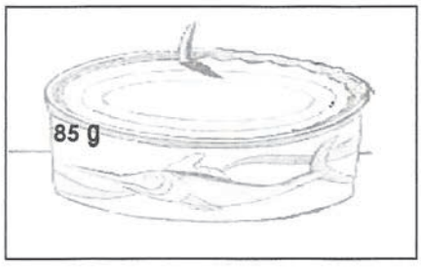
Concepts and skills:

- Introducing formal measuring:
- Comparing, ordering and recording the mass of commercially packaged objects which have their mass stated in kilograms on the packaging, e.g. 2 kilograms of rice, or in grams e.g. 500 grams of salt;
- Learners measuring their own mass in kilograms using a bathroom scale.

What would show evidence of full understanding?

- If the learner indicated that 'grams' is the correct answer: this shows that the learner can read words relating to measurement with understanding and can distinguish between grams and centimetres.
- It also indicates that the learner knows that mass is measured in grams (or kilograms).

18. Circle the correct word in brackets for each of the sentences below.

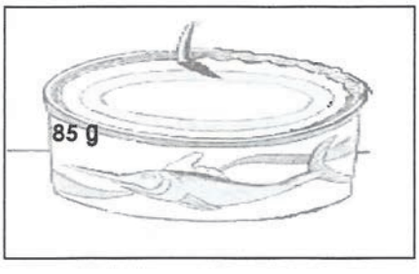


18.1 The mass of the can of fish is measured in (grams; centimetres).

What would show evidence of partial understanding?

- If the learner gave the answer 'centimetres' instead of 'grams' as the answer: this shows that the learner is not yet competent in the standardised units of measurement for mass.

18. Circle the correct word in brackets for each of the sentences below.



18.1 The mass of the can of fish is measured in (grams; centimetres).

What would show evidence of no understanding?

- No response to the question; or
- A response that has no relation to the question.

What do the item statistics tell us?

57% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Learners may confuse mass and length;

- Mathematical vocabulary may not have been consolidated during teaching.

Teaching strategies

- Measurement involves identifying an attribute to be measured (e.g., length, mass, capacity or area) and then using definable, consistent units to find the "how muchness" of the attribute.
- When learners are taught measurement procedures and rules (e.g., formulas) before they understand measurement concepts, they will not fully grasp the meaning of the different attributes, the processes involved in measuring or the significance of the units used to indicate the measures.
- It is important for learners to engage in learning situations that help them to understand different measurable attributes and that teach them to measure those attributes in meaningful ways.
- Learners should also be taught all of the terminology relating to the concepts.

Understanding mass through non-standardised measurement

- Mass refers to the amount of matter in an object.
- Learners often refer to the weight of an object, rather than its mass. Scientifically, weight is a measure of the pull or force of gravity on an object. The weight of an object can vary, depending on its location in space, whereas the mass of the object remains constant.
  - For example, the weight of an object is less on the moon than on the earth, while the mass of the object remains the same.
  - Although learners in the primary grades do not need to distinguish between mass and weight, you can model proper language and can encourage learners to refer to mass rather than weight.

- To assist learners in developing a good concept of mass, give learners tasks that require them to work with a balancing scale to directly compare the mass of objects.
  - Ask learners to order and compare the mass of 3 or more objects by placing pairs of objects on a balancing scale until they have enough information to place the objects in a sequence of heaviest to lightest.
  - Learners should be able to find the mass of objects using informal units of mass on a balancing scale.

- Encourage learners to develop the language (terminology) needed to talk about mass.
  - Mathematical vocabulary relating to mass: Heavy, heavier, heaviest, light, lighter, lightest, mass, kilogram.

- Demonstrate that some objects are lighter or heavier than others by using a balancing scale, e.g. a pencil is lighter than a pencil box. Ensure learners discover that the heavier objects move down and lighter objects move up on the balancing scale.
- Use the correct terminology and make sure that you give the learners the chance to use the

terminology as well.

- Ask learners to use a balancing scale and informal units to measure mass.
- When learners are comfortable using informal units of measurement, you can then expose them to mass in kilograms and grams.

Informal measurement of mass using a balancing scale and non-standard units

- Learners can learn all the principles and practises of measurement using non-standard units.
- Measuring with non-standard units should not be considered to be inferior to measuring with standard units, but is done to teach the concept of mass, not the units of measuring mass.
- Measuring mass with non-standard units involves using a balancing scale and counting how many of the chosen units of measurement have the same mass as the object being measured. For example, a ruler (object being measured) has the same mass as 9 blocks (units of measurement).
  - Ask learners to measure a variety of objects using a range of objects as informal units, for example: stones, ropes, books, apples and marbles.
  - Encourage learners to always state the name of the unit when giving the mass, e.g. the book has the same mass as 34 marbles.
  - Once learners have used an object as a unit of measurement a few times, they should estimate how many of that unit will have the same mass as the object being measured.
  - Estimating before measuring is important, but can only be done once learners have done some measuring with that particular unit.
  - Explain to learners that, in order to compare the masses of different objects, the same unit of measurement needs to be used.
  - For example, if a ruler has a mass of 20 blocks and a pair of scissors has a mass of 20 marbles, you cannot say whether they have the same mass or not, or which one is heavier or lighter.

Practical activity

- Compare three objects using marbles as the units of measurement.
  - Measure the mass of a ruler, a tube of glue and an eraser.
  - Ask 3 different learners to measure the mass of each of the objects using marbles.
  - Ask learners to record their measurements (although measuring is a practical skill learners should record their measurements at all times).

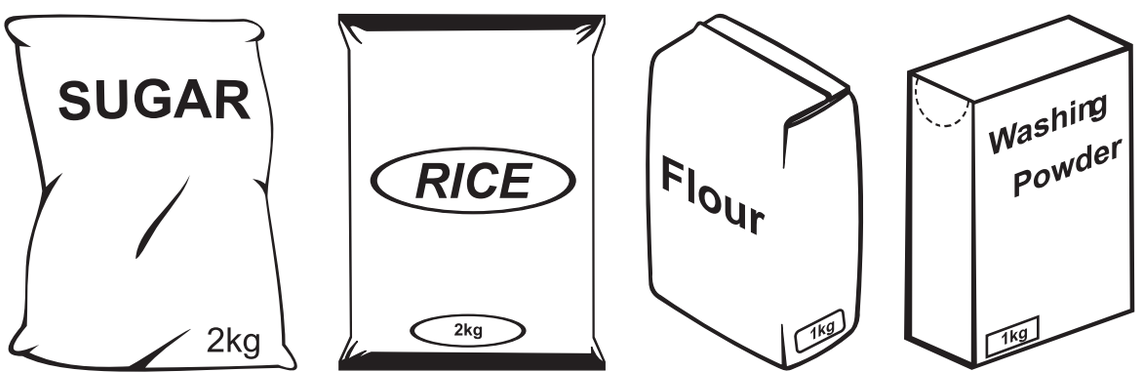
- Ask the learners which object is the heaviest and which is the lightest and discuss the differences.

Understanding mass through standardised measurement

Working with kilograms

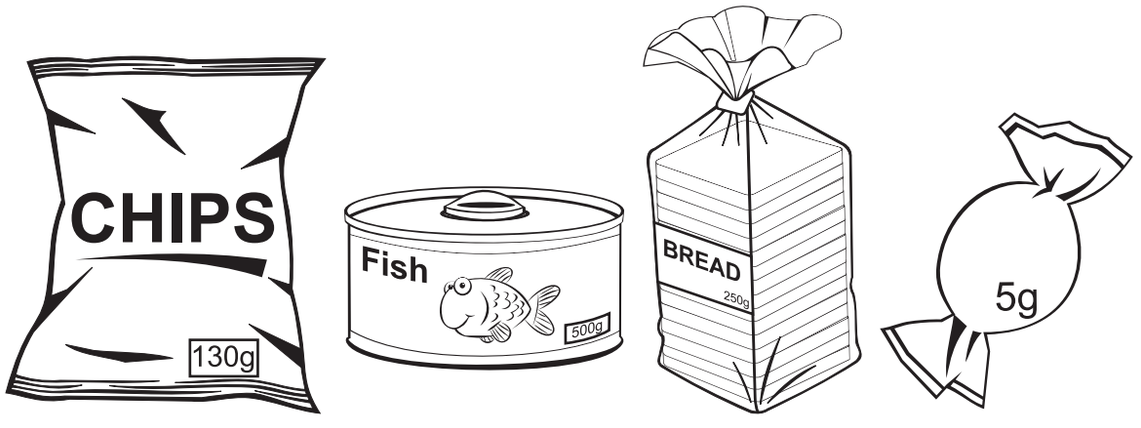
- Introduce learners to kilograms by working with groceries that are sold in kilograms, where the number of kilograms is stated on the packaging.
  - For example: learners can compare the mass of packages of different substances (such as rice, sugar, mealie meal, flour or washing powder) that are sold in 1 kg amounts.
  - Learners can place these on a balancing scale to see that although the size of the packages may differ, they have more or less the same mass.
- Learners can then be given a range of packages of different items to put in sequence from heaviest to lightest, according to the mass stated on the package.

  For example: 10 kg samp, 5 kg mealie meal, 2 kg flour, 2 kg rice, 2 kg sugar,1 kg sugar.



Working with grams

- You can expose learners to grams by using different grocery items where the grams are stated on the packaging.
- Learners can place these items on a balancing scale to see that although the size of the packages may differ the items have more or less the same mass.

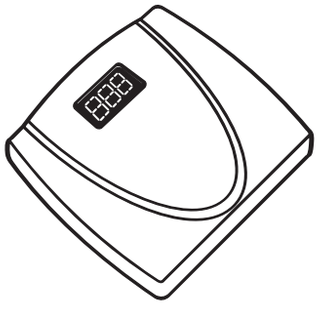


- Once they understand that kilograms are heavier that grams, you can expose learners to the following:
- half of a kilogram is 500 g
- half of 500 g is 250 g
- half of 250 g is 125 g
- four 250 g is the same as 1 kg

Consolidation worksheet **(see printables):** Learners can use the printables to answer questions about the mass of different objects in kilograms and grams.

Practical Activity

- Learners can find their own mass using a bathroom scale.
  - Ask learners to record their measurements, because although measuring is a practical skill learners should record their measurements at all times.
  - Learners can write down their own mass on a piece of paper.



32kg	28kg	22kg	28kg

  - Ask the class to order the masses from the lightest to the heaviest.
  - The masses of learners that are the same can be placed together.

Measuring mass as a context for solving problems and calculations

- During time allocated to *Numbers, Operations and Relationships*, learners can solve problems that use the concept of the informal measurement of mass.

Examples of word Problems

1. Puleka bought 12 kg of mealie meal, 25 kg of sugar and 12 kg of rice. What is the mass of her shopping?

2. Mum bought 32 kg of rice and a 15 kg bag of mealie meal. How much is the mass of her shopping altogether?

Another example of how knowledge of mass can be tested

ANA 2014 Grade 3 Mathematics Item 15

15. The mass of the above washing powder is measured in

_____

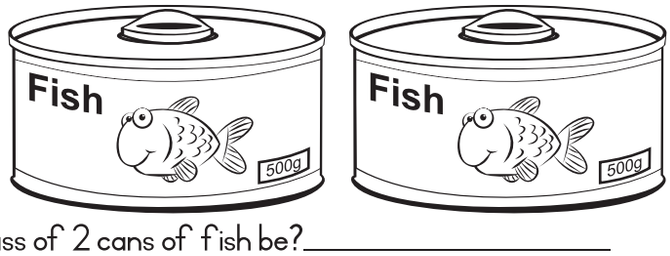
Notes:

Printable: Mass of groceries (1)

1. What is the mass of the sugar?_____

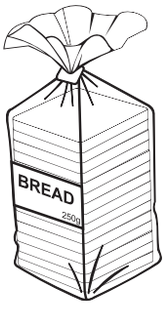
2. What is the mass of the flour?_____

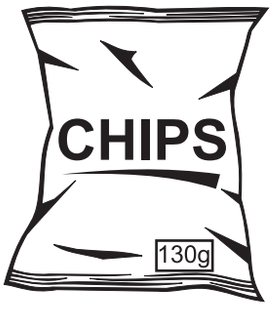
3. What is the mass of the can of fish?_____



4. What will the mass of 2 cans of fish be?_____

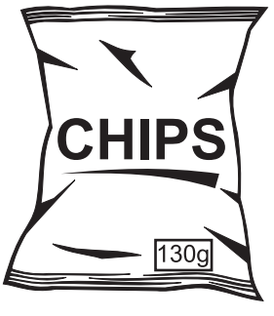
Printable: Mass of groceries (2)



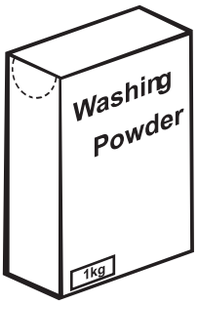


1. Which item is the heaviest? _____

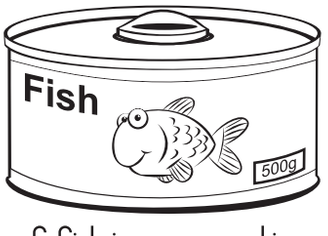
2. Circle the correct answer:



a. The mass of the chips is measured in (kilograms, grams).



b. The mass of the washing powder is measured in (kilograms, grams).

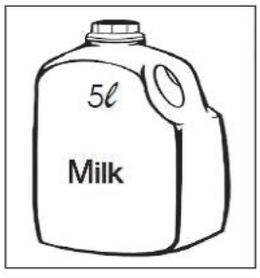


c. The (mass, length) of the can of fish is measured in grams.

Measurement of capacity

ANA 2013 Grade 3 Mathematics Item 18.2

18. Circle the correct word in brackets for each of the sentences below.



18.2. The capacity (content) of the bottle of milk is measured in (metres, litres)

What should a learner know to answer this question correctly?

Learners should be able to:

- Understand the mathematical vocabulary relating to measurement of capacity, e.g. content, capacity, volume, measure, millilitres and litres;
- Know that capacity is measured in millilitres and litres;
- Understand the standardised unit abbreviations of millilitres (ml) and litres (l) by showing understanding of the standardised measuring units;
- Discriminate between measurement of length and capacity.

Where is this topic located in the curriculum? Grade 3 Term 1

Content area: Measurement.

Topic: Capacity.

Concepts and skills:

- Introducing formal measuring;
- Estimate, measure, compare, order and record the capacity of objects by measuring in litres, half litres and quarter litres;
- Compare, order and record the capacity of commercially packaged objects whose capacity is stated in litres or in millilitres on the packaging;
- Know that a standard cup is 250 millilitres;
- Know that a teaspoon is 5 millilitres.

What would show evidence of full understanding?

- If the leaner indicated that 'litre' is the correct answer: this shows that the learner is able to read the words relating to measurement with understanding and can distinguish between metres and litres;
- This also shows that the learner knows that capacity is measured in millilitres or litres.

18.1 Boima ba lekotikoti la tlhapi bo methwa ka (digramo, disentimitara).

18.2 Mothamo wa botlolo ya lebese o methwa ka (dimitara, dilitara).

What would show evidence of partial understanding?

- If the learner indicated 'metres' instead of 'litres': this shows that the learner is not yet competent in the standardised units of measurement for capacity.

18. Etsa sedikadikwe lentsweng le nepahetseng ho a ka masakaneng polelong e nngwe le e nngwe ho tse latelang.

18.2 Mothamo wa botlolo ya lebese o methwa ka (dimitara, dilitara).

What would show evidence of no understanding?

- No response to the question; or
- A response that has no relation to the question.

18. Etsa sedikadikwe lentsweng le nepahetseng ho a ka masakaneng polelong e nngwe le e nngwe ho tse latelang.

18.2 Mothamo wa botlolo ya lebese o methwa ka (dimitara, dilitara).

What do the item statistics tell us?

55% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Learners may confuse mass and capacity with each other.
- Mathematical vocabulary may not have been consolidated during teaching.

Teaching strategies

- Measurement involves identifying an attribute to be measured (e.g., length, mass, capacity or area) and then using definable, consistent units to find the "how muchness" of the attribute.
- When learners are taught measurement procedures and rules (e.g. formulas) before they understand measurement concepts, they will not fully grasp the meaning of the different attributes, the processes involved in measuring or the significance of the units used to indicate the measures.
- It is important for learners to engage in learning situations that help them to understand different measurable attributes and that teach them to measure those attributes in meaningful ways.
- Learners should also be taught all of the terminology relating to the concepts.

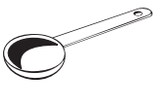
Understanding capacity through non-standardised measurement

- Capacity is the measurement of the amount of liquid or other contents a container can hold.
- Volume is the amount of space an object occupies.
- Help your learners to understand the difference between capacity and volume by using the terminology correctly at all times.
- Mathematical vocabulary relating to capacity: full, empty, half full, half empty, more than, less
- than, as much as, the same as, least, most, litre, millilitre, volume, estimate.

Practical Activities

- A capacity learning centre can help learners to develop an understanding of the capacity of different containers and the knowledge that containers of different shapes may hold the same amount.
- You can set up a capacity learning centre by using a large plastic bin, dry rice or beans and several measuring cups and food storage containers.
- You can place a chart on the wall for learners to record how many scoops of rice or beans fill each container.

Example of the chart:

Fill a 2 litre ice cream container	Measuring cup 	Spoon 	Measuring jug 
Beans	8 cups	80 spoons	2 jugs
Rice	8 cups	80 spoons	2 jugs
Water	8 cups	80 spoons	2 jugs

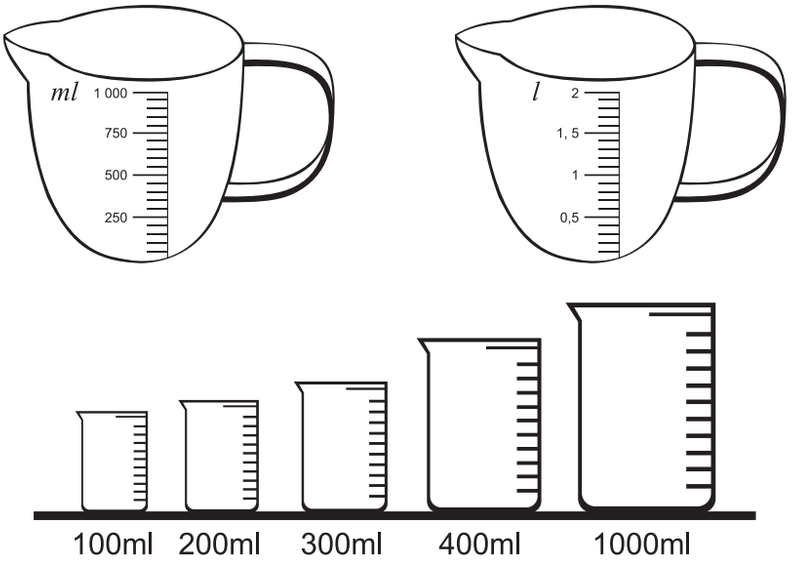
- Review the chart with the children to see which containers hold more, less or the same amount.
- As a follow-up activity, fill a jar with rice or beans.
  - Hold up a cup, and ask the learners to guess (estimate) how many cups of rice or beans are in the jar.
  - The learners can write their estimates down on a piece of paper with their name.
  - The learner who guesses closest to the correct amount can win a small prize.

Understanding capacity through standardised measurement

Working with litres and millilitres

Capacity

- Ask learners if they can remember what units we use to measure capacity. Answer: litres or millilitres
- Practical activity:
  - Show the learners a clearly labelled measuring jug and explain the measuring scales used to determine capacity (e.g. litre, 1 litre, 2 litres).



- Using a set of containers with different capacities, demonstrate how to measure capacity with the measuring jug.
- Say “If I pour liquid from this container into the measuring jug, I will know how much the jug will hold - I will know the jug's capacity! I think (estimate) the jug holds about (approximately, nearly, almost) 500 ml”.
- Using a jug with a numbered 1 litre or 2 litre gradation line, fill it with coloured water so that the water is almost in line with the jug's top gradation marking.
- Guide the learners to read the number on the gradation line closest to where the coloured liquid ends.
- Learners describe the capacity as almost / nearly / close to / a bit more or less / or exactly the number of litres they read off the jug.

Volume

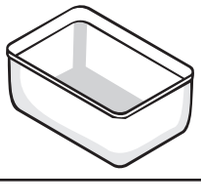
- Fill a jug with different amounts and let the learners read the volume.
- Ensure that the learners practically explore that the following is true:

1 cup	=	250 ml
2 cups	=	500 ml
3 cups	=	750 ml
4 cups	=	1 000 ml
1 000 ml	=	1 litre
500 ml	=	Half of a litre
250 ml	=	Quarter of a litre

- Ask learners to record their measurements, because although measuring is a practical skill learners should record their measurements in writing at all times.

Estimating and measuring capacity

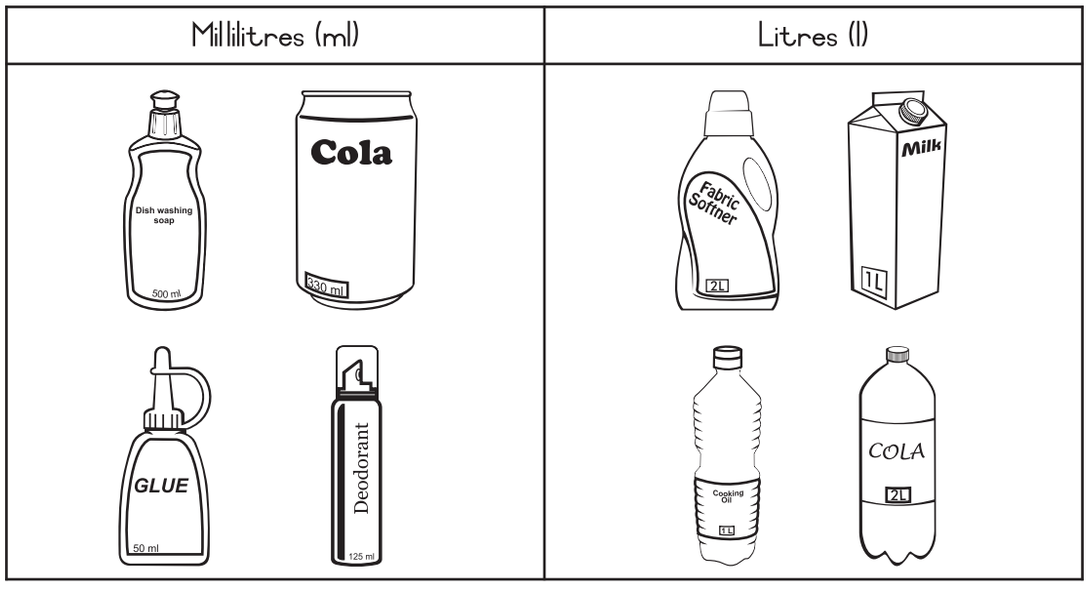
- Give learners a set of containers.
  - Ask learners to estimate the capacity of each container and to record their estimates in a table.
  - After estimating the capacity of each container, ask learners to use the measuring jug to find the actual capacity of each container and to record their measurements **(see printables).**

Container	Estimate the capacity	Actual capacity
	5 l	10 l
	1 l	250 ml
	500 ml	250 ml
	2 l	2 l

Worksheet Activities:

- Provide opportunities for learners to read the capacity indicated in litres or millilitres on pictures of products **(see printables)**.

- Show the learners a range of containers that hold different amounts of liquid.
  - Ask the learners to compare and then order containers from the one that holds the least to the one that holds the most (or vice versa).
  - Ask learners to identify the capacity of each container from its label.
  - After this you can give your learners the comparison worksheet. The learners have to sort the containers into those whose capacity is measured in millilitres and those whose capacity is measured in litres **(see printables).**



Problems and calculations using capacity

- During time allocated to *Numbers, Operations and Relationships* learners can solve problems that use the context of the informal measurement of capacity.

Examples of word problems:

1. Mom buys 4 tubes of hand cream. Each tube contains 55 ml of hand cream. How many millilitres of hand cream are there altogether?

2. Luke has 750 ml of juice in his juice bottle. He drinks 250 ml. How many millilitres are left?

3. A saucepan holds four cups. How many cups will two saucepans hold?

4. A petrol pump can fill 34 cars. How many cars will 2 petrol pumps fill?

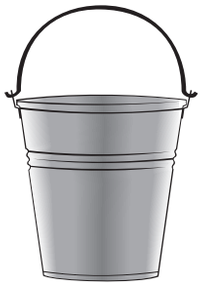
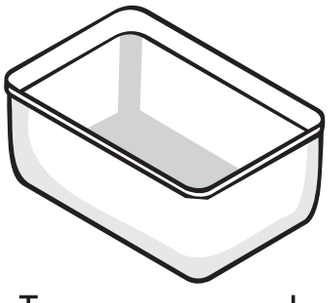
Another example of how knowledge of the measurement of capacity can be tested

ANA 2014 Grade 3 Mathematics Item 16

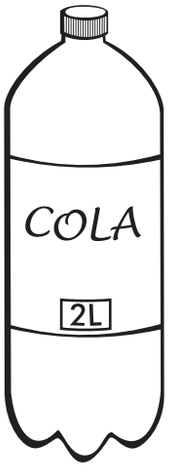
16. The capacity of the above bottle is measured in _____.

Notes:

Printable: Capacity estimation recording sheet

Container	Estimate the capacity	Actual capacity
 Bucket		
 Medicine bottle		
 Cup of tea		
 Ice-cream tub		

Printable: Capacity (1)

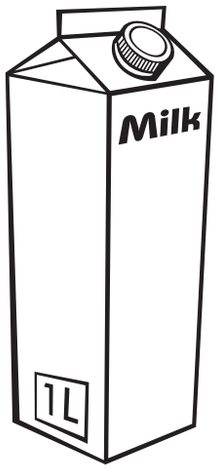


What is the capacity of the cooldrink bottle? _____

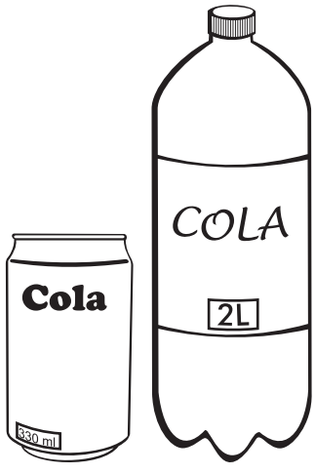
What is the capacity of the can of cooldrink? _____

What is the capacity of the glue bottle?_____

Printable: Capacity (2)



What will the capacity of 2 milk boxes be?_____



Circle the correct answer:

Which container holds the least coke? (Bottle of cola, can of cola)

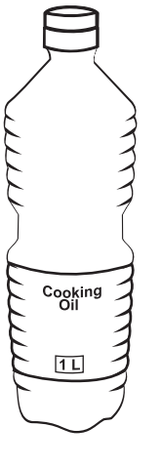
Circle the correct answer:

The capacity of the dish washing soap bottle is measured in (litres, millilitres).

Printable: Capacity (3)

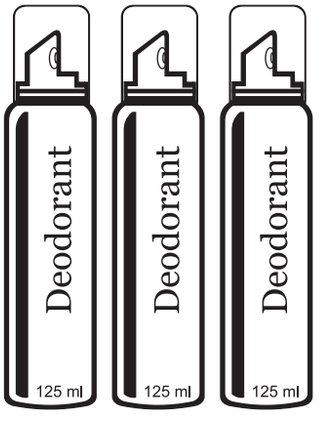
Circle the correct answer:

The capacity of the fabric softener bottle is measured in (millilitres, litres)



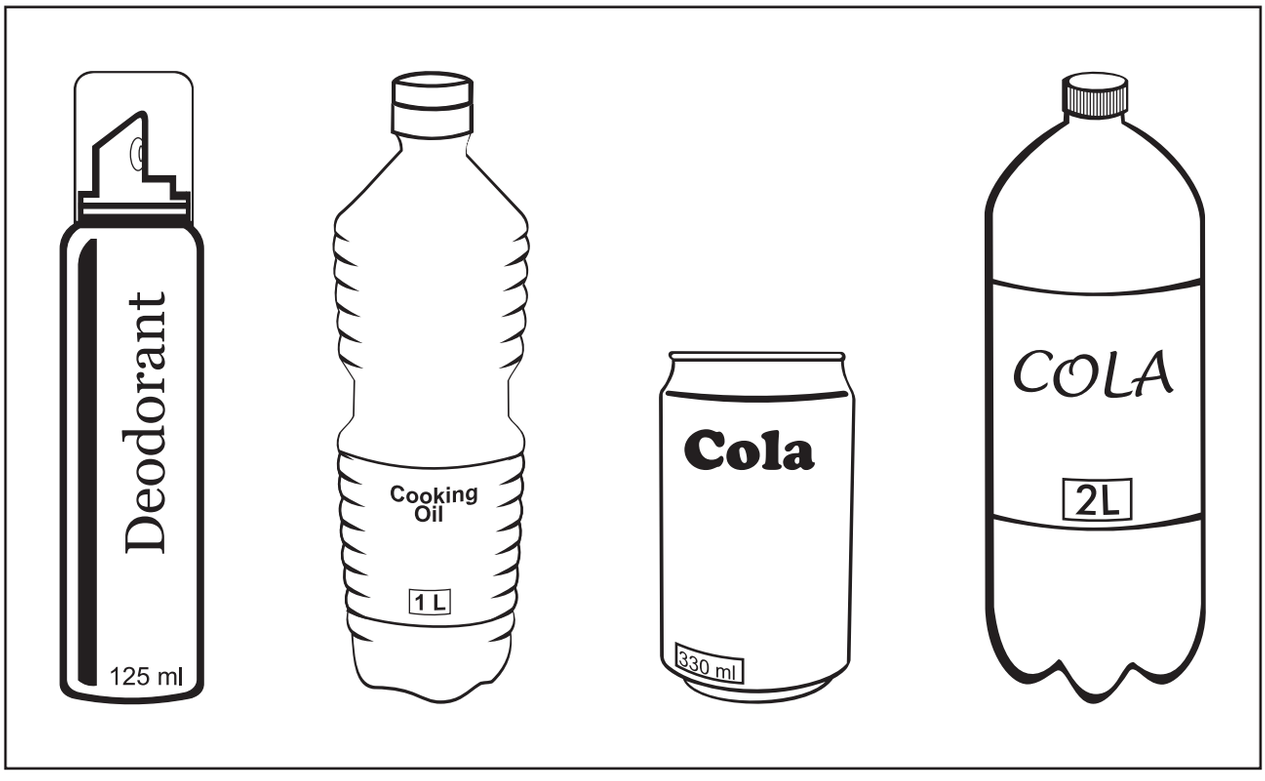
Circle the correct answer:

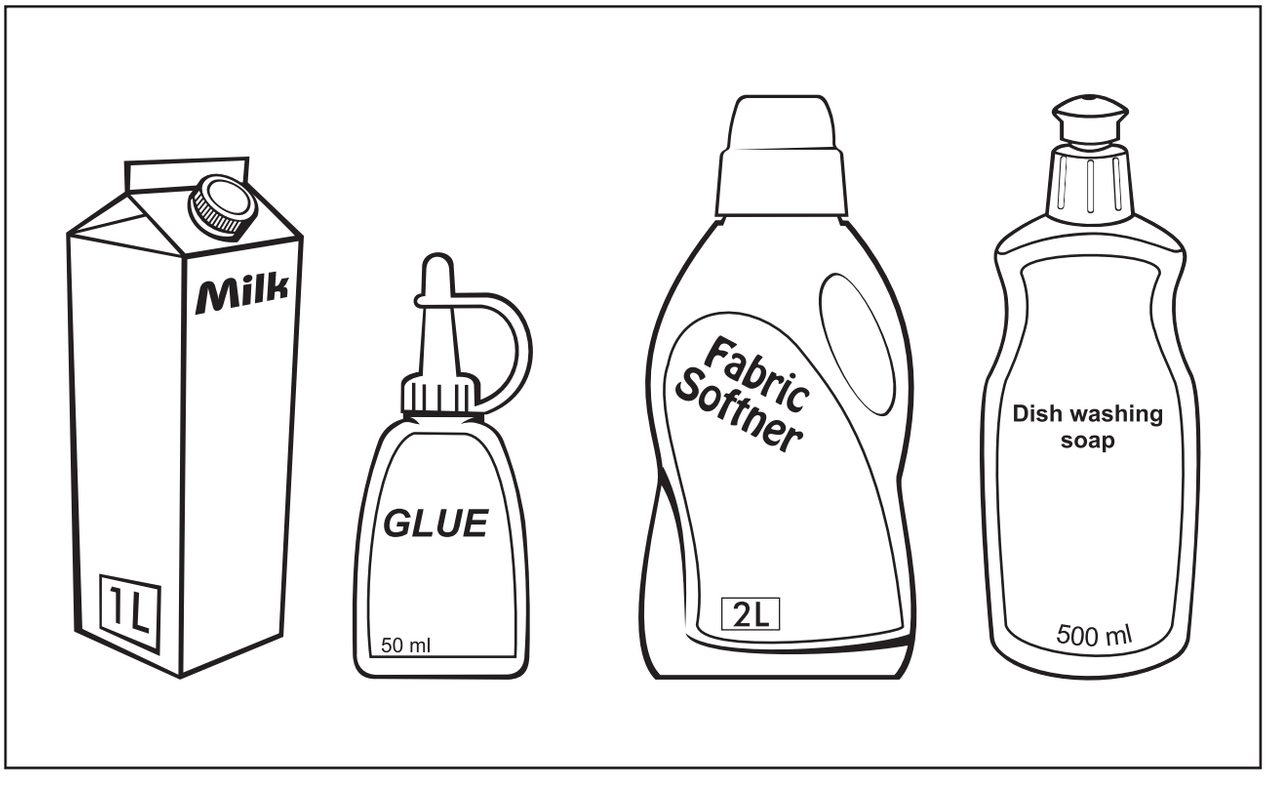
The (mass, capacity) of the oil is measured in litres.



What is the capacity of 3 deodorant spray bottles? _____

Printable: Order according to capacity





Time

ANA 2013 Grade 3 Mathematics Items 5 and 26

5. 9:45 a.m. on an analogue clock shows that the time is

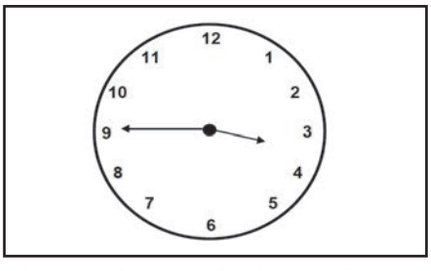
A quarter past ten in the evening

B quarter past ten in the morning

C quarter to ten in the evening

D quarter to ten in the morning

26. Complete the sentence below.



The time on the analogue clock reads _____

What should a learner know to answer these questions correctly?

Learners should be able to:

Item 5

- Read digital time in hours and minutes;
- Know the meanings of a.m. and p.m.;
- Recognise how many minutes are in an hour and a quarter hour.
- Understand the conventions of 'past' and 'to'.

Item 26

- Read analogue time in hours and quarter hours;
- Understand the conventions of 'past' and 'to'.

Where is this topic located in the curriculum? Grade 2 Term 3

Content area: Measurement.

Topic: 4.1 Time.

Concepts and skills:

- Tell 12-hour time in hours and quarter hours on analogue clocks.

Grade 3 Term 3

Content area: Measurement.

Topic: Time.

Concepts and skills:

- Tell 12-hour time in hours and quarter hours on analogue clocks and digital clocks and other digital instruments that show time, e.g. cell phones.

What would show evidence of full understanding?

Item 5

- If the learner gave the answer D (quarter to ten in the morning);
- This shows the learner is able to read digital time in hours and minutes;
- The learner knows that a.m. refers to the morning;
- The learner knows that 9:45 shows fifteen minutes 'to' the next hour.

Item 26

- If the learner gave the answer 'quarter to four', 3:45, 03h45 or 15 minutes to 4;
- This shows that the learner sees that the short hand is in between the three and the four and that the long hand shows a quarter of an hour before the next hour, meaning it is moving 'to' the hour.

What would show evidence of partial understanding?

Item 5

- If the learner chose answer C (quarter to ten in the evening);
- This shows the learner is able to read digital time in hours and minutes, recognising that 9:45 is quarter to ten;
- However, the learner does not understand that a.m. indicates that the time is in the morning.

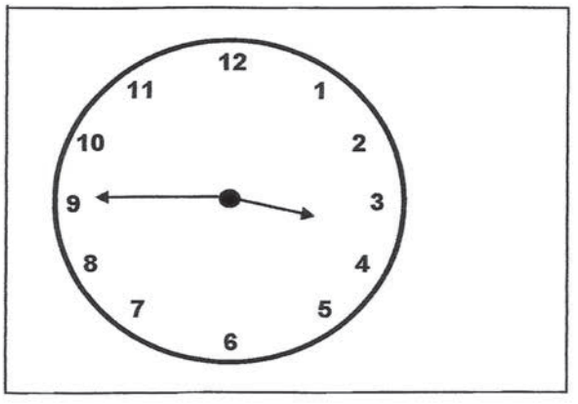
Item 26

- In the example below the learner gave the answer '50 min to 4'.
- This shows two possible thought processes, both showing partial understanding:
- Firstly, the learner may have simply become confused between the numbers 15 and 50. The learner may have written 50 min to 4 instead of 15 min to 4, showing a number concept

misconception rather than a time misconception.

- Alternatively, the learner may have attempted to work out the answer by trying to determine how many minutes past the hour the long hand shows. In this instance, the learner made an error in counting in fives, resulting in the answer 50 minutes instead of 45 minutes.
- The learner is also unsure of the conventions of 'past' and 'to' and identified the time as 50 minutes 'to' 4, rather than 'past' 4.

26. Complete the sentence below.



The time on the analogue clock reads 50 min to 4

What would show evidence of no understanding?

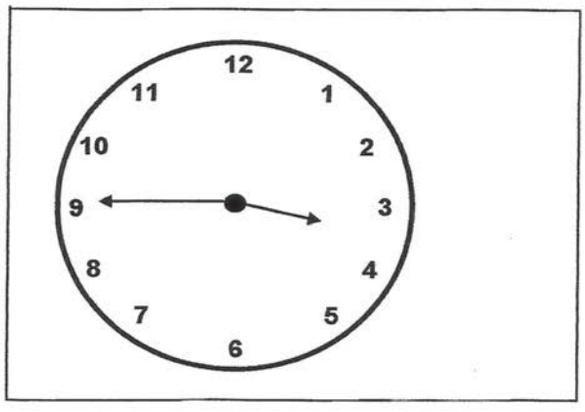
Item 5

- If no answer was given;
- If the learner answered A (quarter past ten in the evening) or B (quarter past ten in the morning): The learner has not understood the conventions of 'past' and 'to' and is unable to read digital time in hours and minutes;
- In giving the answer B (quarter past ten in the morning), the learner may have recognised that a.m. refers to the morning. However, this is a very small component of the question and cannot be considered sufficient to demonstrate even partial understanding.

Item 26

- In the example below the learner gave the answer 3:9 which shows no understanding.
- The learner has not realised that there is significance in the fact that the short hand is pointing in between the three and the four.
- However, the fact that the 3 is written first in the answer 3:9 suggests an understanding that the short hand points to the hour.
- The learner has simply written down the number 9 without an understanding of quarter hours or how many minutes there are in an hour.

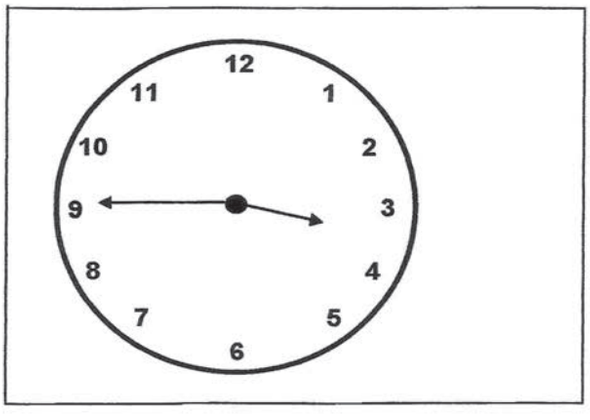
26. Complete the sentence below.



The time on the analogue clock reads 3:9 X

- In the example below the learner gave the answer 9:03 which shows no understanding.
- The learner does not know that the short hand points to the hours and the long hand points to the minutes.
- There is no understanding of how many minutes there are in an hour or of the notion of quarter hours.

26. Complete the sentence below.



The time on the analogue clock reads 9:03

What do the item statistics tell us?

Item 5

38% of learners answered the question correctly.

Item 26

22% of learners answered the question correctly.

Factors contributing to the difficulty of the items

Item 5

- This item has a number of different elements (reading digital time in hours and minutes, knowing how many minutes there are in an hour and understanding a.m., p.m., 'past' and 'to'). Learners may have struggled to achieve accuracy in all of the elements and been confused.

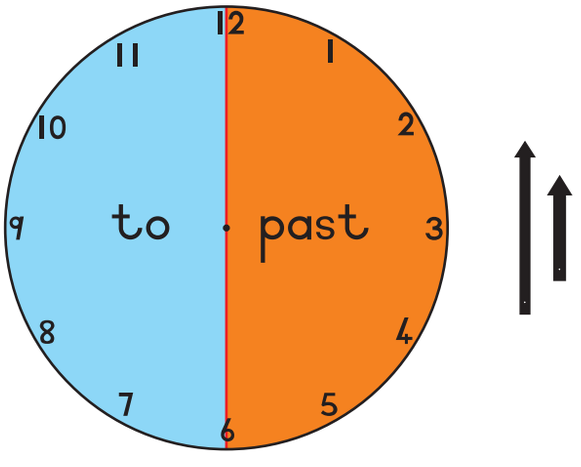
Item 26

- This item may have been made more difficult since the clock face is not a perfect circle and the spacing between the numbers is not the same. This may have misled some learners into thinking incorrectly that the number of minutes between the 12 and the 1 is not the same as the number of minutes between the 1 and the 2.

Teaching strategies

Past and to on the clock

- Provide each learner with a paper plate, a long hand and a short hand made from paper and a split pin. Show them your large, pre-prepared 'past and to clock', using a split pin to attach the hands to the clock face.



- Ask learners to draw a line down the middle of their paper plates, from top to bottom, using a ruler.
- Learners can then colour the right hand side red (or another colour) and the left hand side blue (or another colour), as seen in your large example.
- Explain to the learners that the line dividing the two colours tells us when the long hand is moving 'past' the hour and when it is moving 'to' the next hour. Explain to the learners that:
  - When the long hand is moving through the red half of the clock face, then it is moving 'past' the hour.
  - When the long hand is moving through the blue half of the clock face, then it is moving 'to' the next hour.

- Ask learners to write the numbers on the clock, taking care to write them in the correct positions.

- The 12 needs to be written at the top, on the dividing line between the red and blue halves.
- The 6 needs to be at the bottom, on the same dividing line.
- The 3 and the 9 need to be midway between the 12 and the 6, on the right and the left hand sides respectively.
- The other numbers can be written into the remaining spaces, trying to keep the numbers the same distance apart.

- Help the learners to attach the long and short hands to the clock face using the split pin so as to enable the hands to move around the clock face.
  - Ask the learners to position the long hand on the 12 and the short hand on the 3. Ask the learners "What is the time?" The learners should respond "3 o'clock".
  - Ask the learners to move the long hand into the red half of the clock face, showing them on your large clock as an example. Ask the learners "What can you tell me about the long hand?" Learners should respond "The long hand is moving past 3 o'clock".
  - Ask the learners to move the long hand past the 6 and into the blue half of the clock face. Ask the learners "What can you tell me about the long hand?" Learners should respond "The long hand is moving to the next hour or "The long hand is moving towards 4 o'clock".

Half hours and quarter hours

Half hours

- Ask learners to work in pairs or groups of four, using the paper plate clocks made in the previous activity.
- Ask learners to look at their clock faces and to describe what they see. Learners should respond "I see a circle / I see numbers / I see two halves", etc.
- Encourage learners to notice that the red and blue parts of the clock face divide it exactly in half.

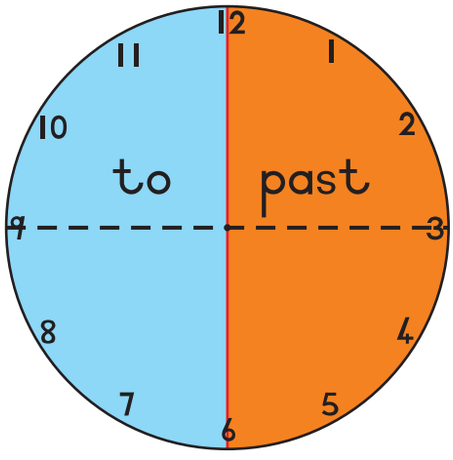
- Ask learners "If the long hand is on the 6, how far has it moved around the clock face?" Learners should respond "It has moved halfway around the clock face".
- Explain to learners "This is why we say it is 'half past' the hour, because the long hand has moved halfway around the clock face".

- Ask learners to place the long hand on the 6 and the short hand halfway between the 1 and the 2. Ask them "What time is it?" Learners will respond "Half past 1".
- Ask the learners "How did you know what the hour was?" Learners will respond that "The short hand is halfway between the 1 and the 2, which tells us that it is moving past 1 o'clock, and towards 2 o'clock".

Quarter hours

- On your large clock, draw a line from the 9 to the 3, as shown below.

- Ask the learners to draw a line from the 9 to the 3 on their paper plate clock faces as well.
- Ask learners to look at their clock faces and to describe what they see. Learners should respond “I see a circle / I see numbers / I see four quarters etc”.
- Encourage learners to notice that the drawn line divides the two halves of the clock face into quarters.



- Ask the learners “If the long hand is on the 3, how far has it moved around the clock face?” Learners should respond “It has moved a quarter of the way around the clock face”.
- Encourage learners to realise that the long hand is in the red part of the clock, so it is said that the long hand is now a “quarter past” the hour.
- Explain to learners “This is why we say it is 'quarter past' the hour, because the long hand has moved a quarter of the way around the clock face”.

- Ask learners to place the long hand on the 3 and the short hand between the 5 and the 6.
- Ask them “What time is it?” Learners should respond “Quarter past 5”.
- Ask the learners “How did you know what the hour was?” Learners should respond that “The short hand is between the 5 and the 6, which tells us that it is moving past 5 o'clock and towards 6 o'clock”.

- Ask the learners “If the long hand is on the 9, is it in the blue or the red part of the clock?” Learners should respond “It is in the blue part of the clock face, so it is moving to the next hour”.
- Ask the learners “How much of the clock face is left for the long hand to move through before it gets to the next hour?” Encourage the learners to realise that there is a quarter of the circle left for the long hand to move through in order to get to the next hour.
- Explain to learners “This is why we say it is 'quarter to' the hour, because the long hand has to move a quarter of the way around the clock face before it gets to the next hour”.

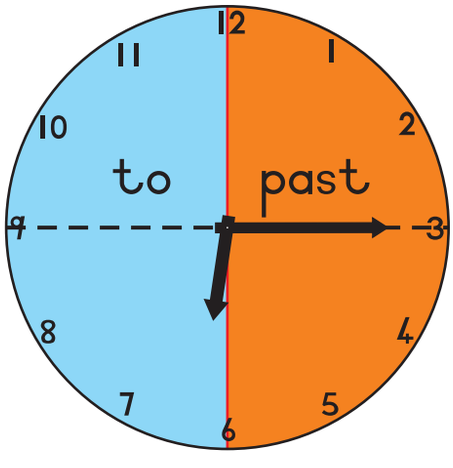
- Ask the learners to place the long hand on the 9, and the short hand between the 10 and the 11.
- Ask them “What time is it?” Learners may respond with either “Quarter to 10”, or “Quarter to 11”.

- Encourage the learners to notice that the short hand is between the 10 and the 11. Remind them that this tells them that the long hand is moving 'past' 10 o'clock and 'to' 11 o'clock.
- Explain to the learners that this is why we say it is 'quarter to 11".

Matching digital time and analogue time

- Ask learners to work in pairs or groups of four, using their paper plate clocks from the previous two activities.
- Ask learners "How many minutes are there in an hour?' Learners should respond by saying "There are 60 minutes in an hour".
- Explain to the learners that they can see this by counting in 5s as they point to each of the numbers on the clock face.
- Ask learners to start with their fingers on the 12. Ask them "When we start counting in 5s, where must our fingers be?" The learners may say either "On the 12" or "On the 1".
- Explain to the learners that it takes five minutes for the long hand to move from the 12 to the 1, so we can only start counting in 5s when we move our finger from the 12 to the 1.
- Count with the learners in 5s (5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60) as you point to the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 on your large clock. (Learners point to the numbers on their paper plate clocks).
- Ask learners "How many minutes are there in half an hour?" Learners should respond "30 minutes". Allow learners to point to the numbers, counting in 5s if necessary.
- Ask learners "How many minutes are there in a quarter of an hour?" Learners should respond "15 minutes". Allow learners to point to the numbers, counting in 5s if necessary.

- Write the time '6:15' on the board.
- Ask learners to show you this time on their paper plate clocks.



- Ask the learners "How did you know to show the time like this?" Learners should respond by saying "The '6' of '6:15' tells me the hour, so I put the short hand on the 6. Then the '15' tells me the long hand has moved 15 minutes past the hour, so I counted in 5s to the 3 and put the long hand on the 3.

I then moved the short hand to between the 6 and the 7 because the long hand was moving past the hour, so it couldn't be on the 6 exactly".

Other examples of how knowledge of time can be tested

ANA 2014 Grade 3 Mathematics Item 4

3:15 a.m. on an analogue clock shows that the time is...

A quarter past three in the morning.

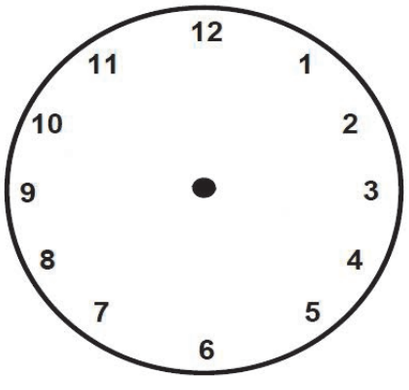
B quarter past three in the evening.

C quarter to three in the morning.

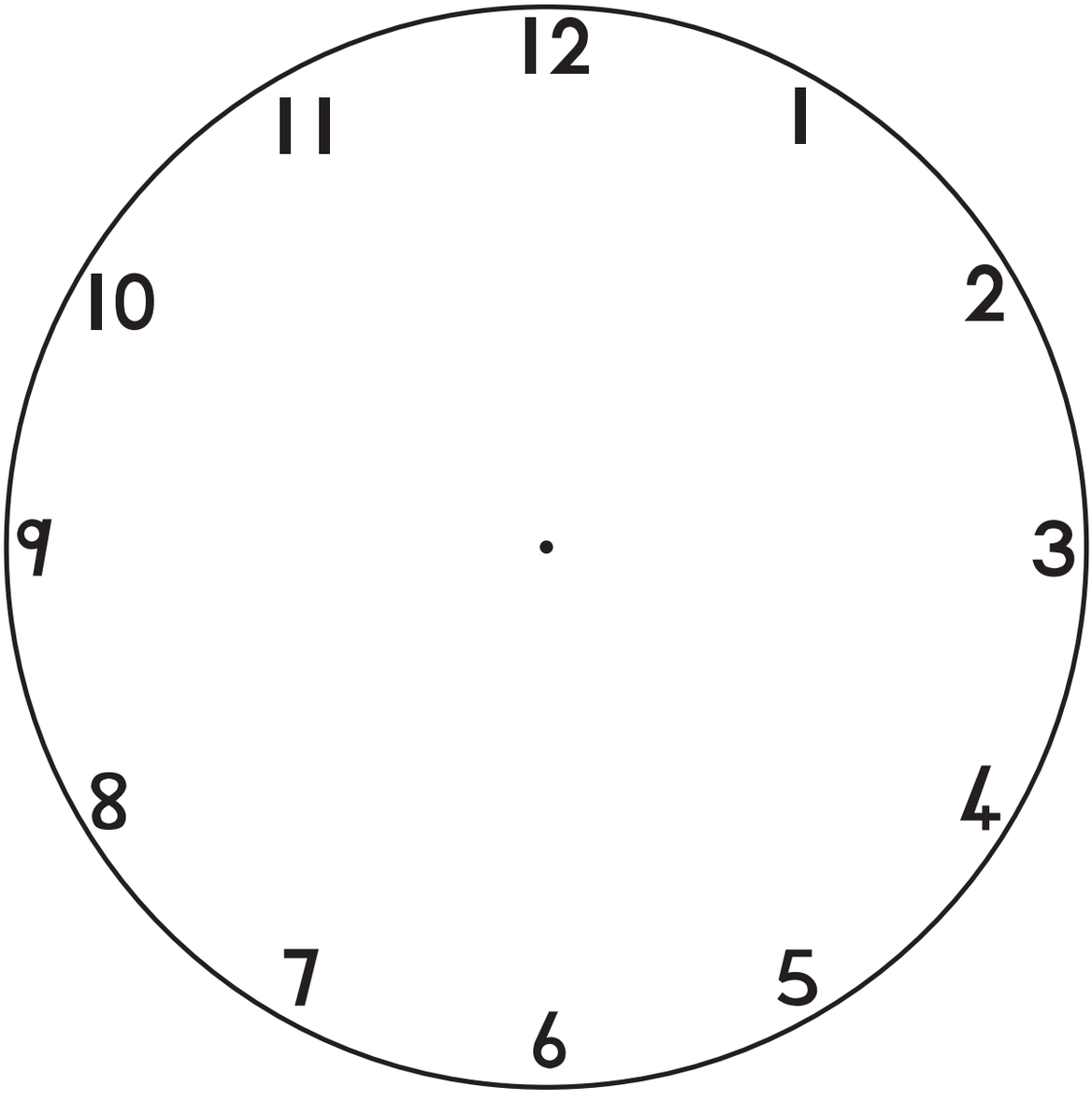
D quarter to three in the evening.

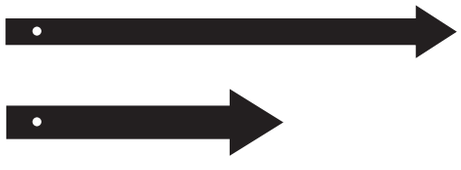
ANA 2014 Grade 3 Mathematics Item 27

27. Draw the hands on the analogue clock to show that the time is 05:15.



Printable: Clock face





Position

ANA 2013 Grade 3 Mathematics Items 22.1 and 22.2

22. Help the puppy to find his way to his kennel.

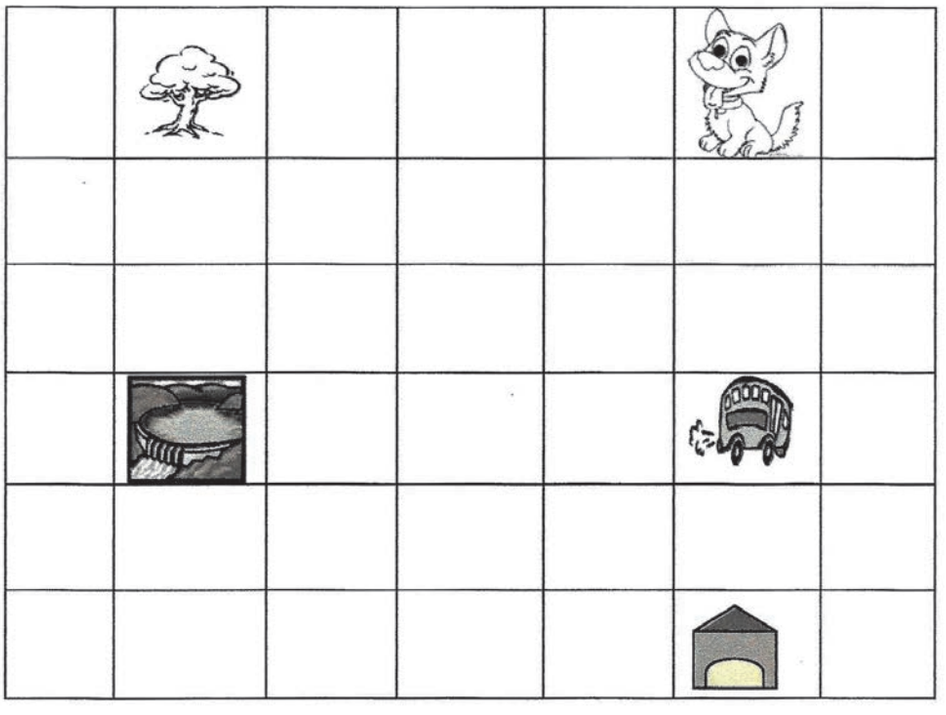
The puppy runs to the tree.

He feels thirsty and runs to the dam to drink water.

He runs to the bus and then to his kennel.

22.1 Draw arrows in the grid to show how he ran.

22.2 How many blocks did he run altogether?



What should a learner know to answer these questions correctly?

Learners should be able to:

- Read given directions in order to establish a route on an informal map;
- Move through blocks in a straight line, turning left or right to reach the pictures identified in the directions;
- Count each consecutive block used in the established route so as to determine how far the puppy

travelled in order to reach his kennel;

- Recognise that the pictures on the informal map need to be included as part of the route and counted as a step in the pathway.

Where is this topic located in the curriculum? Grade 3 Term 3

Content area: Shape and space.

Topic: Position, orientation and views.

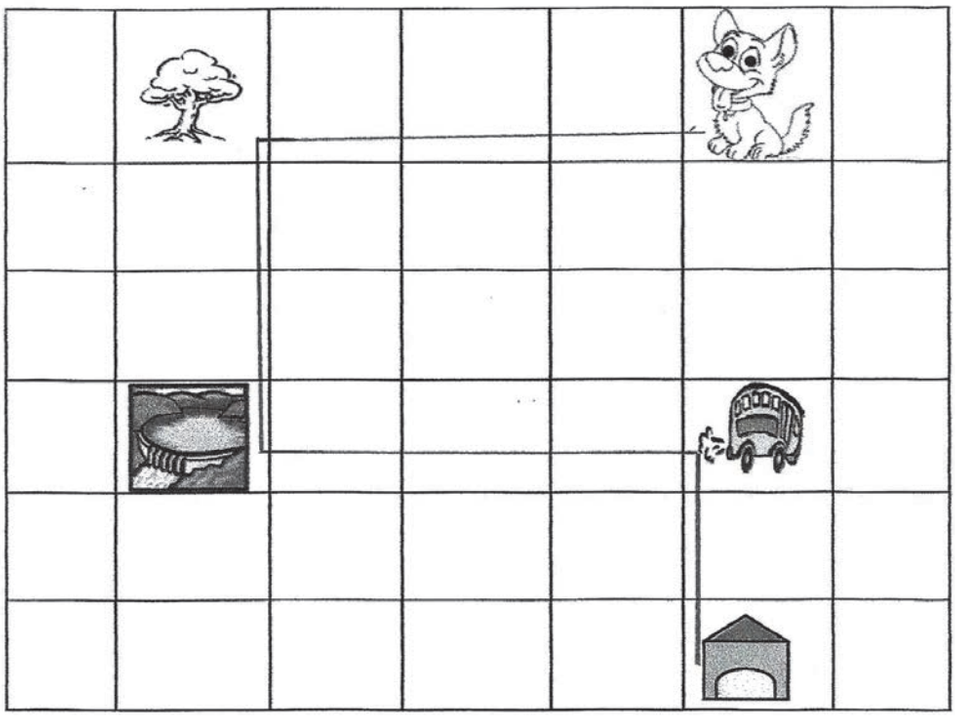
Concepts and skills:

- Follow directions from one place to another on an informal map.

What would show evidence of full understanding?

Item 22.1

- If the learner provided the response as illustrated: this shows the learner understood that there needed to be one route chosen in order for the puppy to reach the kennel.
- The learner was able to move towards each of the pictures listed, changing direction so as to move through the blocks in a straight line.



Item 22.2

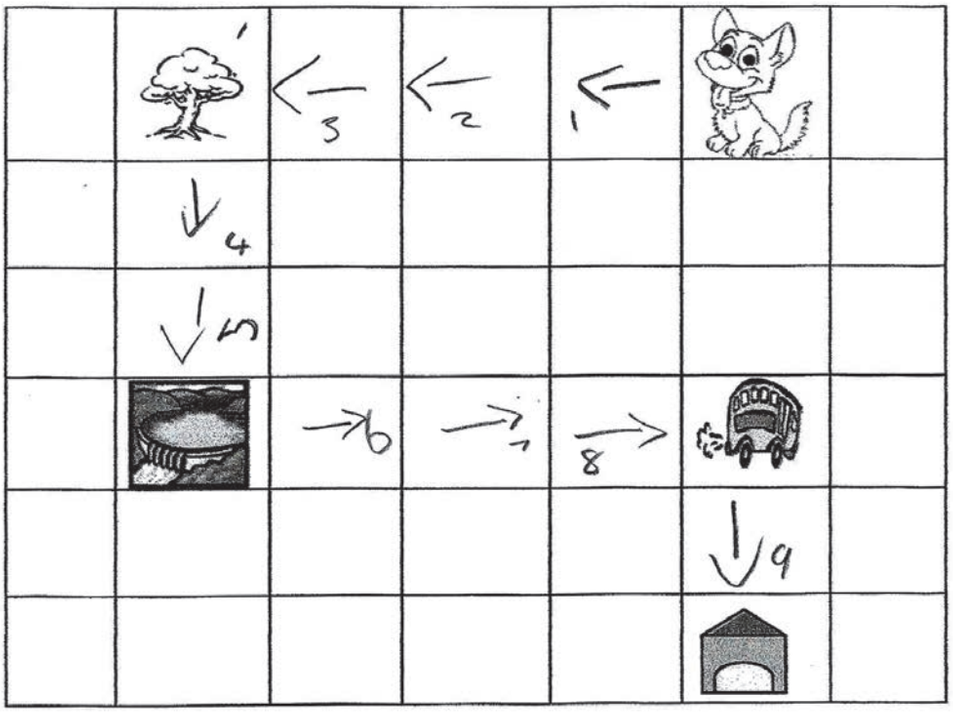
- If the learner answered 13: this shows the learner realised that each block in the path needed to be counted, regardless of whether the block had a picture in it or not.

22.2 How many blocks did he run altogether? 13

What would show evidence of partial understanding?

Item 22.1

- If the learner provided the response in the following illustration: this shows that the learner understood the directions and was able to follow the correct route, but turned before the picture of the dam and then needed an extra turn to get onto the correct row to get to the bus.
- The learner also turned before the picture of the bus and then needed an extra turn towards the kennel.



Item 22.1

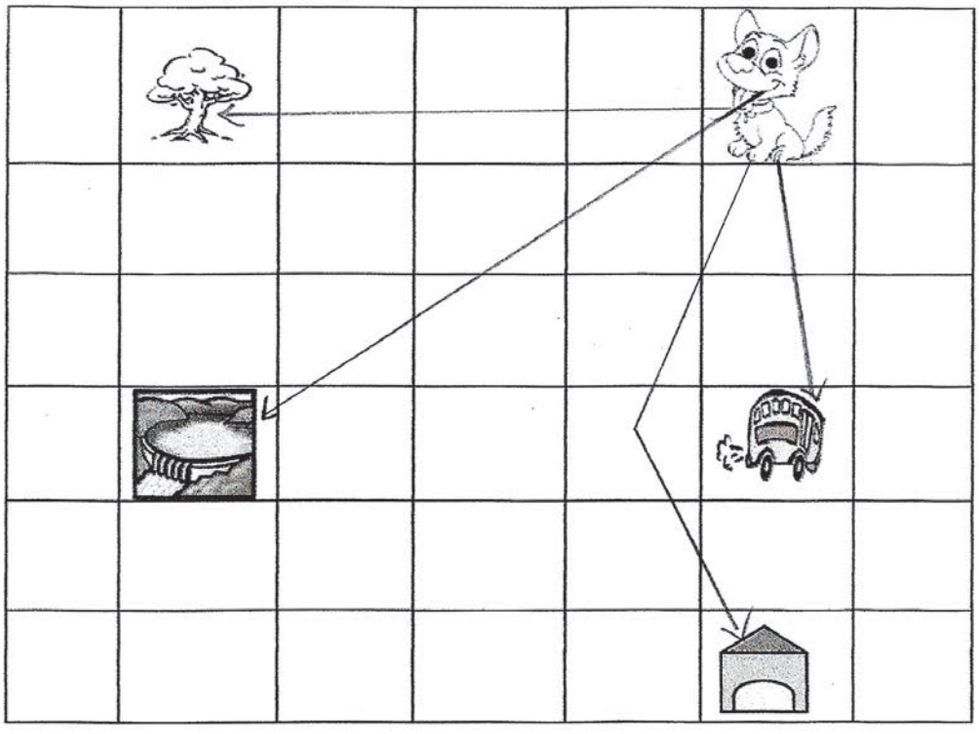
- If the learner answered 9: tis shows that the learner understood that the blocks of the path that the dog followed had to be counted, but did not count the blocks with pictures, which resulted in the response being four blocks less than the expected answer.

22.2 How many blocks did he run altogether? 9blocks

What would show evidence of no understanding?

Item 22.1

- The learner who provided the following response showed that he/she did not understand that he/she had to find a route to the kennel by following a path from picture to picture; the learner drew four paths which all orignated from the same point and which ended at each of the pictures; and the learner did not understand that all the pictures needed to be included in one route.



Item 22.2

- If the learner gave the answer 4: this shows the learner did not understand that each block in the route should have been counted; the learner counted the four paths taken from the original starting point instead.

22.2 How many blocks did he run altogether? 4

What do the item statistics tell us?

Item 22.1

55% of learners answered the question correctly.

Item 22.2

31% of learners answered the question correctly.

Factors contributing to the difficulty of the items

- The learners may not have realised that they had to count the picture blocks as part of the route. For example:
  - The learners may have thought that they needed to turn at (before) the dam, as opposed to counting the picture of the dam block as part of the pathway.
  - This may have seemed logical as the picture of the dam is situated in a direct line with the previous picture in the pathway.
  - Learners may have thought that they could not 'step' into the block as the dam is too close to the

edge.

- In addition, Item 22.2 is dependent on Item 22.1, which means that if a learner provided an incorrect response for 22.1, then the learner's response for 22.2 would most likely be incorrect as well.

Teaching strategies

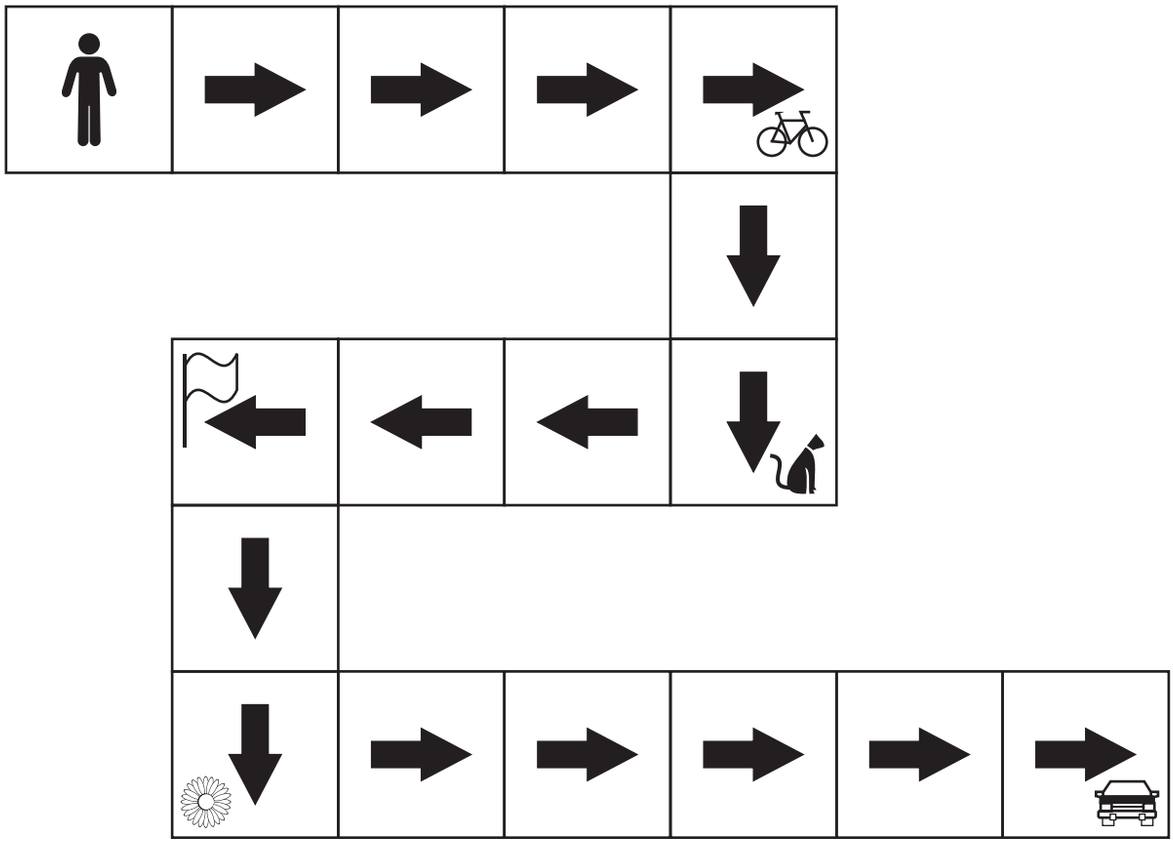
Follow the clues

- This activity needs to be well planned beforehand, with careful consideration of the facilities and context of your school.
- Plan a short, simple route through the school grounds.
- Prepare a list of no more than five directions using visible land marks.
- For example:
  - Start at the classroom door and walk to the staircase;
  - From the staircase, walk to the dustbin;
  - From the dustbin, walk to the office;
  - From the office, walk to the school hall;
  - From the school hall, walk to the fence.
- The directions need to be simple sentences, containing only one instruction.
- The directions need to take the learners through the school or playing fields in a logical progression.
- Initially it may be necessary to do this activity as a whole class, where you lead the learners along the route whilst they follow in orderly lines.
- With practice and experience, depending on your context, it may become possible to send learners out to follow the clues in small independent groups.
- Ask learners to stand in two straight lines and to find a partner in the line next to them. Ensure that each learner has a partner.
- Provide each pair with the list of directions.
  - Ask learners to discuss the first direction in their pairs. Ask “Where do you need to go?” Learners should respond “To the staircase”.
  - Walk with the class to the staircase and stop there to discuss the next direction.
  - Ask learners to discuss the second direction in their pairs. Ask “Where do you need to go?” Learners should respond “To the dustbin”.
  - Ask learners “Why can't we go straight to the office?” Encourage learners to realise that they cannot skip steps in the route and that they need to follow the directions in the order that they are given.
  - Walk with the class to the dustbin and stop there to discuss the next direction.
  - Ask learners to discuss the third direction in their pairs. Ask “Where do you need to go?” Learners should respond “To the office”.
  - Ask learners “Why can't we go straight to the fence?” Encourage learners to realise that they cannot skip steps in the route and that they need to follow the directions in the order they are given.

- Walk with the class to the office and stop there to discuss the next direction.
- Ask learners to discuss the fourth direction in their pairs. Ask “Where do you need to go?” Learners should respond “To the school hall”.
- Walk with the class to the school hall and stop there to discuss the next direction.
- Ask learners to discuss the fifth direction in their pairs. Ask “Where do you need to go?” Learners should respond “To the fence”.
- Walk with the class to the fence and stop there to discuss the path that they walked.
- Ask the learners “What did you do on your walk?”. Learners should respond that they walked from one point to another as they followed the directions that they were given.
- Ask learners if they skipped out any of the directions, or if they went to the same landmark (for example: dustbin) more than once. Learners should respond “No, we followed the directions in order and we only went to each landmark once”.

Instruction discussion

- Draw the following diagram on the board:



- Ask learners to work in pairs or groups of four and provide them with scrap paper or white boards.
- Explain to learners that the diagram shows the path that Lindo walked to get to his mom's car.
  - Ask learners “Where do you think Lindo's path starts?” Learners should respond that Lindo's path starts where the picture of the boy is because the arrows show where he walked.
  - Ask learners to discuss Lindo's path amongst themselves and to talk about what he saw while he

walked.

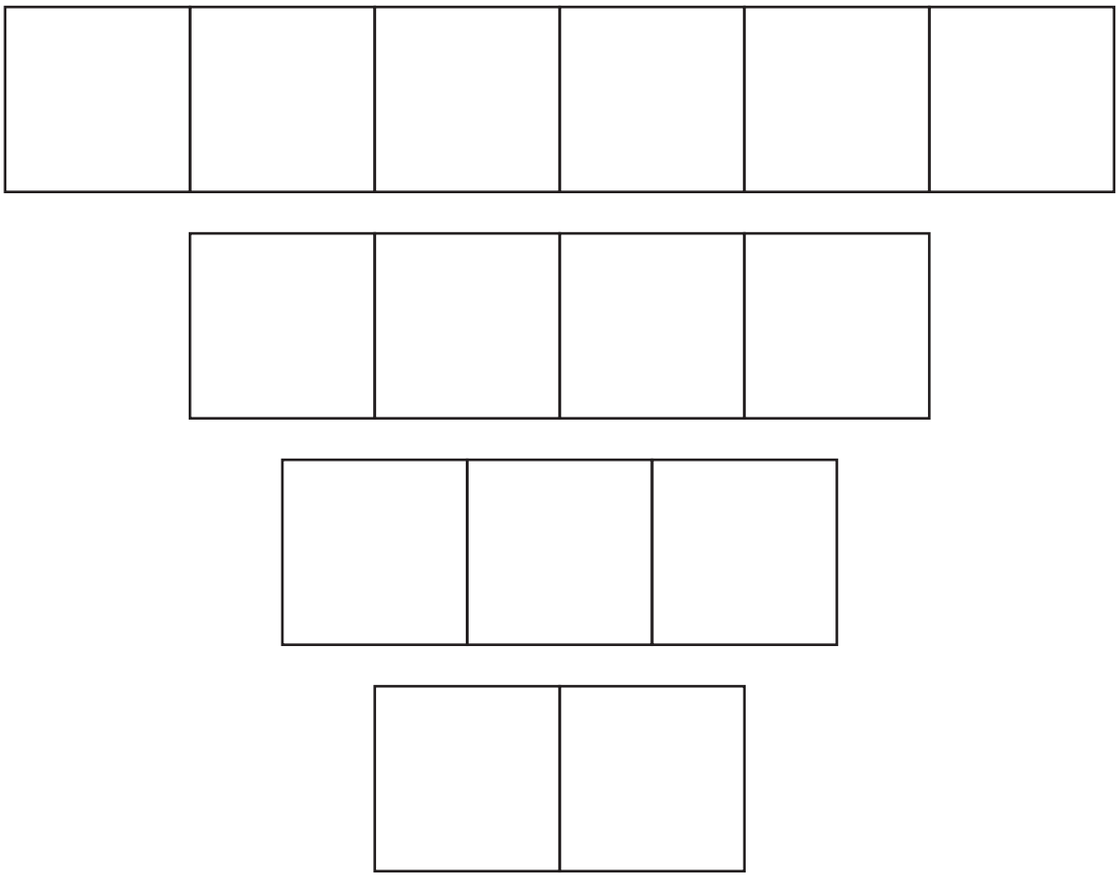
- Ask learners “What did Lindo do first?” Learners should say “Lindo walked to the bicycle”.
- Ask learners what they notice about the arrows going to the bicycle. Learners should say “All the arrows face the same way”.
- Ask learners to count how far Lindo walked to get to the bicycle. Ask “How many blocks did you count?” Learners may say “3” because they do not count the block containing the bicycle picture. In this case, tell learners that that they need to count the block that the bicycle is in as well. Ask learners to count all the blocks with arrows up to and including the bicycle. The learners should count 4.
- Ask learners “Where did Lindo go next?” Learners should say “Lindo walked from the bicycle to the cat”.
- Ask learners what they notice about the arrows going to the cat. Learners should say “All the arrows face the same way”.
- Ask learners to count how far Lindo walked to get to the cat. Ask “How many blocks did you count?”
- Encourage the learners to realise that Lindo walked 2 blocks because they need to count the block that contains the cat as well.
- Ask learners, “Why didn't you count the block that the bicycle is in as well?” Learners should say “We already counted that block so we can't count it again”.
- Ask learners “Where did Lindo go next?” Learners should say “Lindo walked from the cat to the flag”.
- Ask learners what they notice about the arrows going to the flag. Learners should say “All the arrows face the same way”.
- Ask learners to count how far Lindo walked to get to the flag. Ask “How many blocks did you count?”
- Encourage the learners to realise that Lindo walked 3 blocks because they need to count the block that contains the flag as well.
- Ask learners “Why didn't you count the block that the cat is in as well?” Learners should say “We already counted that block so we can't count it again”.
- Ask learners “Where did Lindo go next?” Learners should say “Lindo walked from the flag to the flower”.
- Ask learners what they notice about the arrows going to the flower. Learners should say “All the arrows face the same way”.
- Ask learners to count how far Lindo walked to get to the flower. Ask “How many blocks did you count?”
- Encourage the learners to realise that Lindo walked 2 blocks because they need to count the block that contains the flower as well.
- Ask learners “Why didn't you count the block that the flag is in as well?” Learners should say “We already counted that block so we can't count it again”.
- Ask learners “Where did Lindo go next?” Learners should say “Lindo walked from the flower to

the car".

- Ask learners what they notice about the arrows going to the car. Learners should say "All the arrows face the same way".
- Ask learners to count how far Lindo walked to get to the car. Ask "How many blocks did you count?"
- Encourage the learners to realise that Lindo walked 5 blocks because they need to count the block that contains the car as well.
- Ask learners "Why didn't you count the block that the flower is in as well?" Learners should say "We already counted that block so we can't count it again".

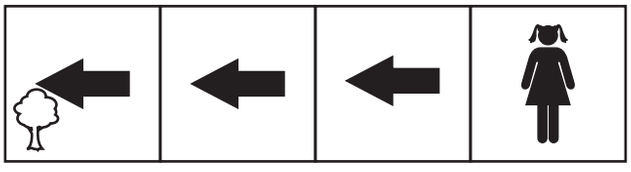
Make a route

- Ask learners to work in pairs or groups of four.
- Provide each pair or group of four with four strips of blocks as shown in the diagram that follows:



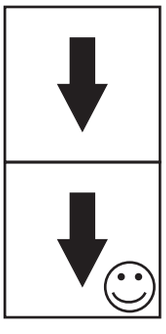
- Explain to learners that they are going to use their block strips to create the path that Nabeelah walks on her way home from school each day. Tell the learners that:
  - Nabeelah walks to the tree;
  - Then Nabeelah walks to her friend's house;
  - Then Nabeelah walks to the stop sign;
  - Then Nabeelah walks to her house.

- Ask learners to hold up their block strips that have 4 blocks.
- Ask learners to place this strip on their desks horizontally.
  - Ask learners to put their fingers on the first block on the right hand side of the strip.
  - Ask a learner from each pair or group of four to draw Nabeelah in the first block on the right hand side.
  - Ask learners to put their fingers on the first block on the left hand side.
  - Ask a learner from each pair or group of four to draw a tree in the first block on the left hand side.
  - Ask learners to then draw arrows in each block as shown in the next diagram:



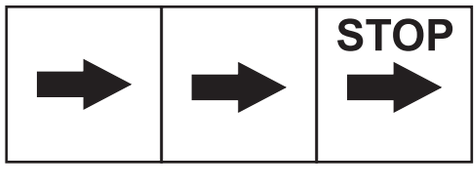
- Ask learners to count how many blocks Nabeelah walked. Learners should respond “3”.
- Ask learners why they did not say 4. Learners should reply that they could not count the block where Nabeelah started because she had not started walking yet. They could only count from the second block as that was when she started walking.

- Ask learners to hold up the 2 block strip.
  - Ask learners to place this strip on their desks vertically under the tree in the four block strip.
  - Ask learners to draw a smiley face in the bottom block, to show that Nabeelah walks to her friend's house.
  - Ask learners to then draw arrows in each block as shown below:



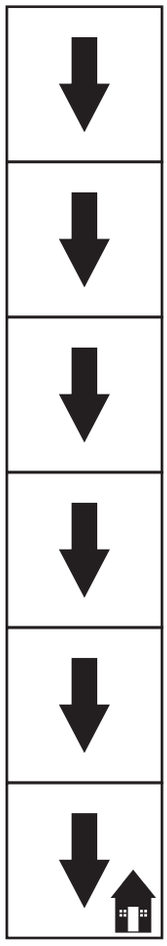
  - Ask learners to count how many blocks Nabeelah walked. Learners should respond “2”.
  - Ask learners why they did not say 3. Learners should reply that they could not count the block where the tree is because they have already counted that block.

- Ask learners to hold up the 3 block strip.
  - Ask learners to position it horizontally next to the smiley face in the two block strip.
  - Ask learners to write the word 'stop' in the last block on the right hand side, to show that Nabeelah walks to the stop sign.
  - Ask learners to then draw arrows in each block as shown below:

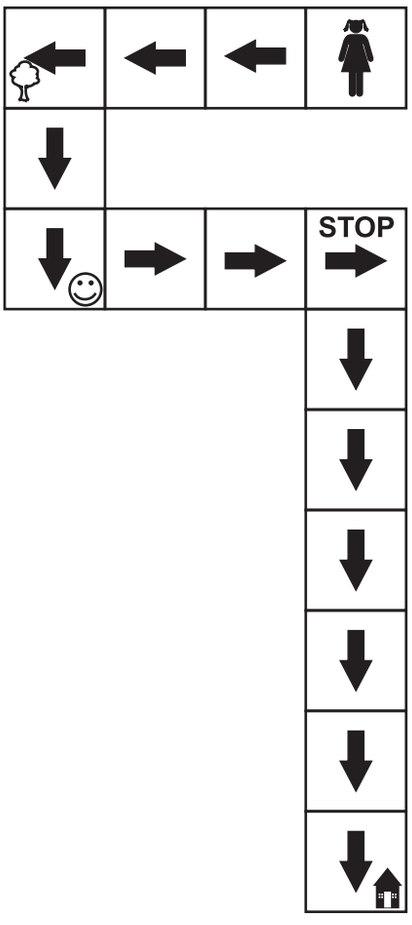


- Ask learners to count how many blocks Nabeelah walked. Learners should respond, “3”.
- Ask learners why they did not say 4. Learners should reply that they could not count the block where the smiley face is because they have already counted that block.

- Ask learners to hold up the 6 block strip.
  - Ask learners to position it vertically under the block with the stop written in it.
  - Ask learners to draw a house in the bottom block, to show that Nabeelah walks to her house.
  - Ask learners to then draw arrows in each block as shown below:



- Ask learners to count how many blocks Nabeelah walked. Learners should respond “6”.
- Ask learners why they did not say 7. Learners should reply that they could not count the block where the word 'stop' is because they have already counted that block.
- Ask learners to look at their completed maps (as shown in the next diagram):



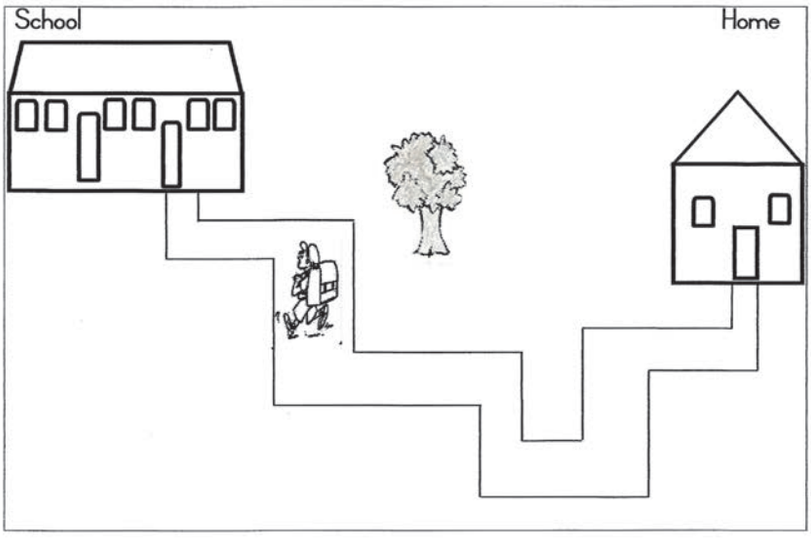
- Ask learners how many blocks Nabeelah walked in total. Learners should respond by saying “14”.
  - Ask learners “How did you get to '14'?” Learners should say that they needed to count each block, including the ones with the pictures.

- Alternatives:
  - You can use this example as it is, or make up other routes using strips of blocks.
  - Ask learners to make up their own routes with strips and tell their own stories to go with the routes. They can work in pairs. Each learner creates a route and then explains it to their partner.

Another example of how knowledge of position can be tested

ANA 2014 Grade 3 Mathematics Item 20

20. Look at the picture and answer the questions below.



20.1 How many turns does Thato take from home to school?

_____

20.2 When Thato walks to school, will the tree be on his left or right?

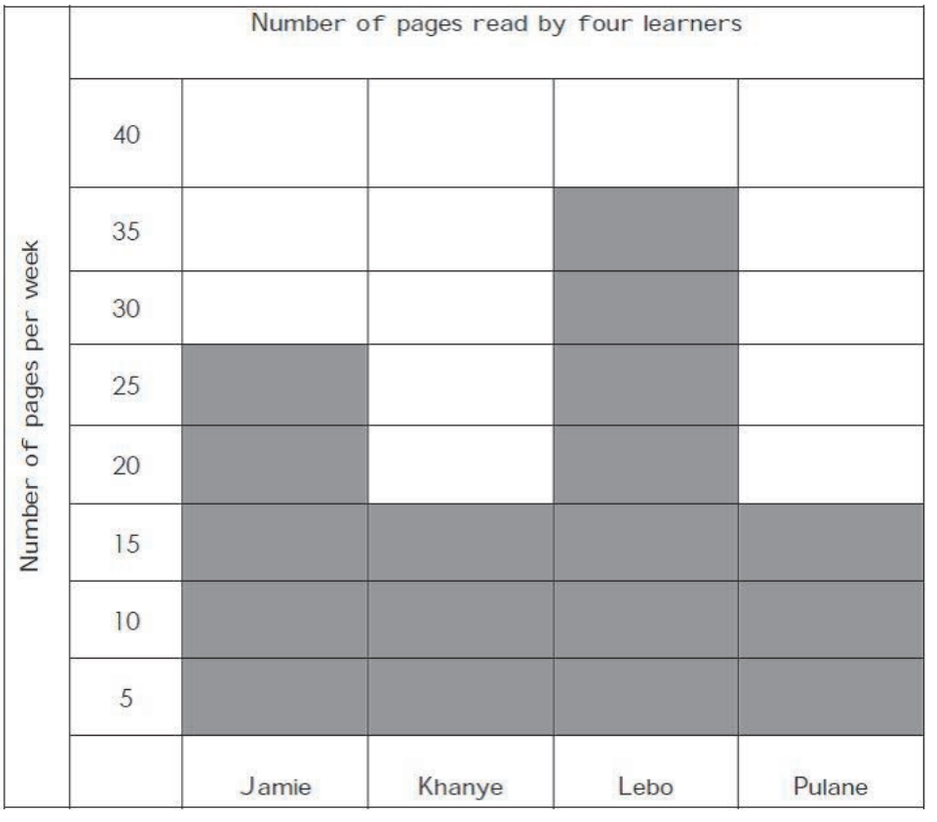
_____

Notes:

Data handling: graph interpretation

ANA 2013 Grade 3 Mathematics Items 23.1 and 23.2

23. Study the graph and then answer the questions that follow.



23.1 Which learners read an equal number of pages? _____

23. 2 Who read the most pages? _____

What should a learner know to answer these questions correctly?

Learners should be able to:

- Read the questions with understanding;
- Analyse and interpret the data in the graph correctly;
- Understand the name of the graph;
- Understand that a graph involves categories and counts of the number of people or of items;
- Identify if a graph is vertical or horizontal in structure.
- Understand the specific mathematical vocabulary involved: study, equal number, most.

Where is this topic located in the curriculum? Grade 3 Term 2 - 3

Content area: Data handling.

Topic: Analyse and interpret data.

Concepts and skills:

- Answer questions about data presented in pictographs;
- Bar graphs.

What would show evidence of full understanding?

- If the learner gave the correct answers to both questions: this shows understanding of the mathematical vocabulary and correct interpretation of a graph.

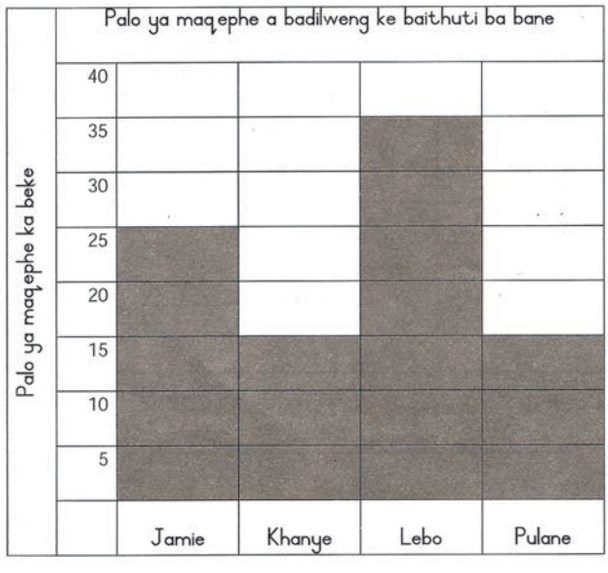
Item 23.1

- If the learner gave the correct answer it shows that the learner understands the mathematical vocabulary "equal".

Item 23.2

- If the learner answered the question correctly it shows that the learner understands the mathematical vocabulary "most".

23. Bala kerafo mme o arabe dipotso tse latelang.



23.1 Ke baithuti bafe ba badileng palo e lekanang ya maqephe ka beke?

Ke Khanye le pulane

23.2 Ke mang ya badileng maqephe a mangata ho feta ba bang?

Ke lebo

What would show evidence of partial understanding?

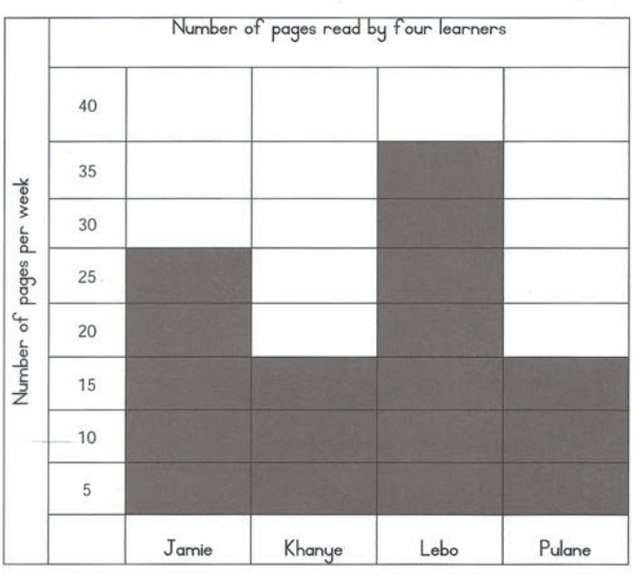
Item 23.1

- If the learner gave the names of two learners as the answer, but not the correct two names, the learner understood that "which learners" indicates the answer is two or more names, but did not understand the mathematical vocabulary of "equal number" or could not read the graph correctly.

Item 23.2

- If the learner gave a learner's name as the answer, but not the correct one, the learner understood that the answer should be the name of a learner, but could not read the graph correctly. The learner might not have understood the word "most".

23. Study the graph and then answer the questions that follow.



23.1 Which learners read an equal number of pages?

Jamie and Pulane ✓

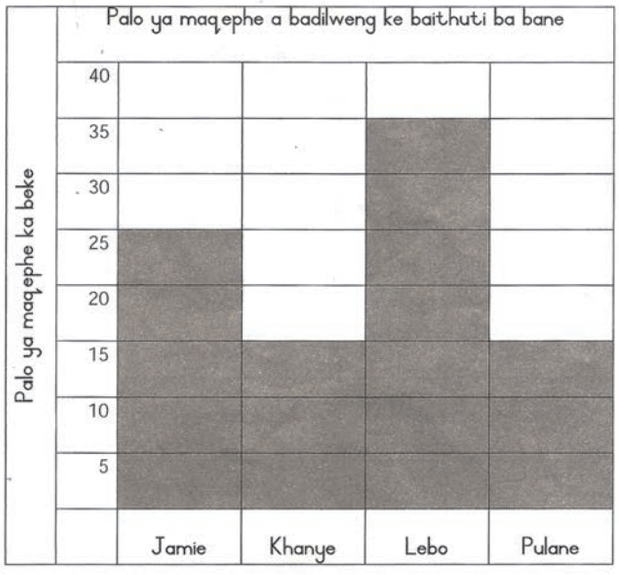
23.2 Who read the most pages?

Lebo ✓

What would show evidence of no understanding?

- If the learner gave incorrect answers to both the questions;
- If a numeric answer was given to either of the questions: the learner did not understand that the names of learners were required for the answers and not numbers from the graph - the graph had to be used to find out the correct names in order to answer the questions;
- If no attempt was made to answer the question.

23. Bala kerafo mme o arabe dipotso tse latelang.



23.1 Ke baithuti bafe ba badileng palo e lekanang ya maqephe ka beke?

23

23.2 Ke mang ya badileng maqephe a mangata ho feta ba bang?

5

What do the item statistics tell us?

Item 23.1

61% of learners answered the question correctly.

Item 23.2

74% of learners answered the question correctly.

Factors contributing to the difficulty of the items

Item 23.1

- Learners may not understand the correct mathematical meaning of an equal number;

Learners find graph interpretation difficult.Factors contributing to the difficulty of the item:

- Learners may not understand the correct mathematical meaning of most.
- Learners had difficulty with graph interpretation.
- Learners did very well in this item which shows that skills and concepts such as “most” assessed in this item are understood.

Teaching strategies

Teaching data handling by starting with collecting and organising data

- Data handling is a process and learners should understand that you can:
  - Collect and organise data;
  - Represent data;
  - Analyse and interpret data.
- Data handling is a contextualised topic and it can be done in the context of number, length, capacity, mass, time or money.

- First discuss with your learners that there are many ways of collecting data:
  - **Observing things –** record what you see, e.g. how many different kinds of trees are there in the school grounds?
  - **Asking questions –** collect information by asking questions, e.g. What colour toothbrushes do learners have?
    How do the learners get to school?
    What do the learners like or dislike?
  - **Conducting a survey** - ask some people the same questions using a questionnaire, e.g. do a survey to find out how many hours people watch TV per day.
  - **Experimenting** - trying something out or counting or measuring to discover something, e.g. counting how many different vehicles pass the school during a specific time period.

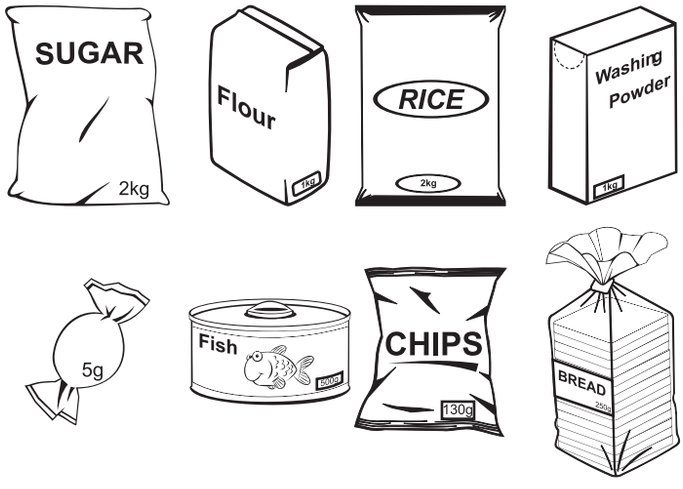
- Examples of objects or information that learners can collect to sort and organise are:
- Kinds of footwear children like;
  - How much different family members weigh;
  - Different lengths of clothes, e.g. scarf, pants, socks;
  - Kinds of cleaning materials used at home;
  - Different kinds of food;
  - Time spent watching TV;
  - Objects that can float/sink;
  - What objects are made of, e.g. wood, glass, plastic;
  - Favourite colours, toothpaste, soap, fruit, vegetables or sport;
  - Ages of family members;
  - Ages of learners in the class;
  - Weather chart: how many sunny/windy/rainy days;
  - Labels in clothes: tumble dry/hand wash/dry clean;
  - Farm animals, wild animals, pets;
  - Types of transport to school;
  - Birthdays of learners/family;
  - Different kinds of homes;

- Toys learners have;
- Reading lists of learners.

- After collecting data or doing a survey, the information must be organised into tables or lists or records without numbers.
- Information or objects collected should be sorted or organised according to one or more attribute.

Activity

- Learners should collect empty containers from different kinds of food.
- Containers can be sorted according the unit their contents are measured in: grams (g) or kilograms (kg).



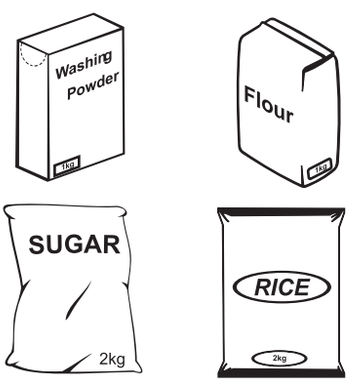
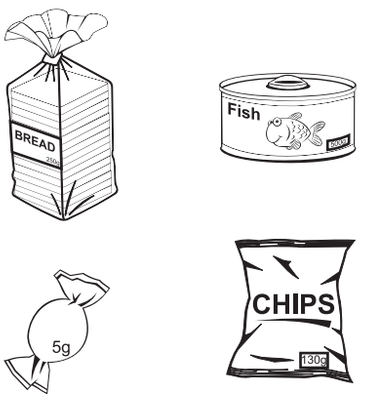
- To complete the data handling process learners must also represent the data.

Teaching data handling by representing data in different ways

- Grade 3 learners are expected to be able represent data in the following ways:
  - Lists
  - Tables
  - Tallies
  - Pictographs
  - Bar graphs
- After the data has been sorted and organised, questions based on the data should be asked to assist the learners with data analysis and interpretation.

Examples of data represented in different ways:

Recording without numbers

killogram (kg)	gram (g)
	

- Learners represent the gram and kilogram containers in a record without numbers.

List

- Containers can be sorted (by name) according to the unit their contents are measured in: grams (g) or kilograms (kg).

killogram (kg)	gram (g)
• Washing powder • Flour • Sugar • Rice	• Bread • Fish • Sweet • Chips

Table

- Containers can be sorted (by number) according to the unit their contents are measured in: grams (g) or kilograms (kg).

Container	1	2	3	4	Total
kg	√	√	√	√	√
g	√	√	√	√	√

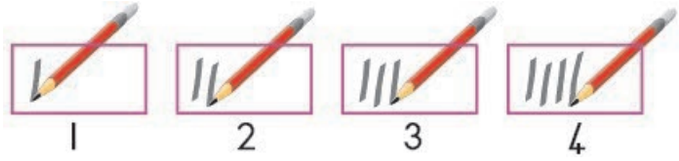
(√ is one item)

Tally

- Learners must understand how tallying works and it should be taught to learners in a practical way.
- Explain to learners that when they tally they must count in multiples of 5, representing the counting

in a specific manner:

- Learners make tally marks for 1 to 4:



- The fifth mark is made across the previous 4 marks:

- Then the learners continue making single marks again:

- Remember: Every fifth mark is drawn across the previous 4 marks. This makes it easy to see the total later on!

Example: A tally of 12

- You can see there are two 5s (making 10) plus another 2 singles, making **12.**

- You can use tally marks when doing a survey**.**
  - Example: "What is your favourite colour?"
  - Draw a table with the names of the colours.
  - Ask some learners what their favourite colour is.
  - As the learners answer, put a mark next to the colour they name.
  - Tally the responses for each colour in the table.
  - When you are done tallying, write the totals into the table.
  - Result: 4 learners liked yellow, 5 liked red, 6 liked blue, only 1 liked green and 4 learners liked pink.
  - Questions must be asked based on the tallies to help the learners analyse and interpret the data.

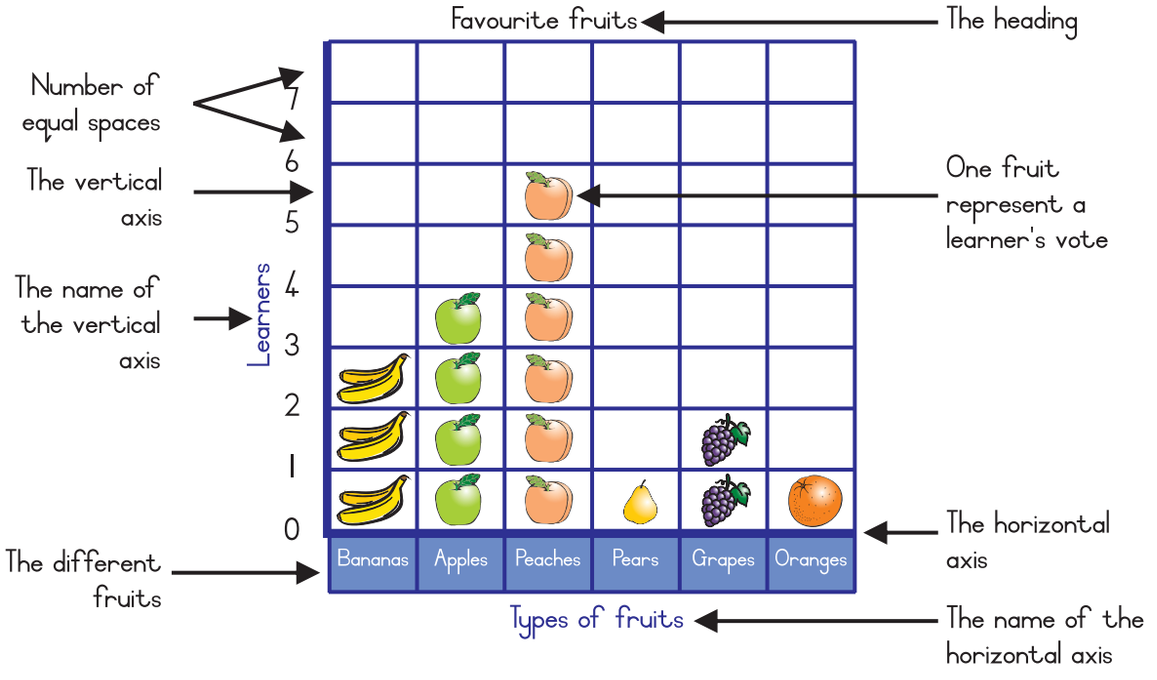
Yellow	////	4
Red	////	5
Blue	//// /	6
Green	/	1
Pink	////	4

Pictograph and bar graph

- Learners must be aware of the following mathematical expressions and understand their meanings:
  - Graph: A diagram showing the relationship between some variable quantities. Graphs make it easy for us to compare numbers.
  - Data: The complete set of individual pieces of information which is being used.
  - Frequency: The frequency of data is the number of times each piece of that data is found.
  - Pictograph: A pictograph is used to display the count in each category and each count is represented by a picture. The layout of the graph can be vertical or horizontal.
  - Bar graph: A bar is used to display the count in each category. The layout of the graph can be vertical or horizontal.

Pictograph

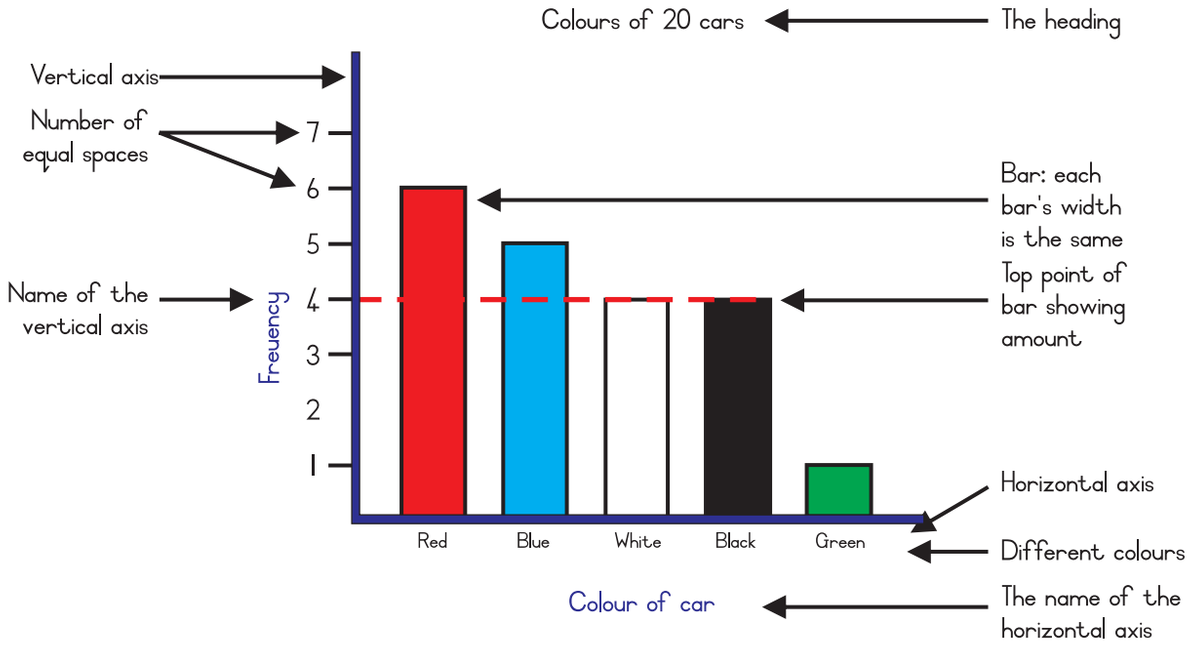
- Explain the layout of a pictograph to the learners.



- A pictograph must have the following:
  - A heading;
  - A vertical axis with a name;
  - A horizontal axis with a name;
  - Symbols indicating the items being counted.
- One picture in this pictograph represents one fruit. This is the key. One picture could represent more than one item in a pictograph.
- Ask questions such as:
  - What is the least favourite fruit?
  - What is the favourite fruit?
  - How many more peaches are there than grapes? etc.
- Allow your learners to draw their own pictographs and also to interpret some that have been drawn for them **(see printables)**.

Bar graph

- Explain the layout of a bar graph to the learners using different examples.
- Go step by step through the bar graph and assist the learners if they do not feel confident creating their own bar graphs.

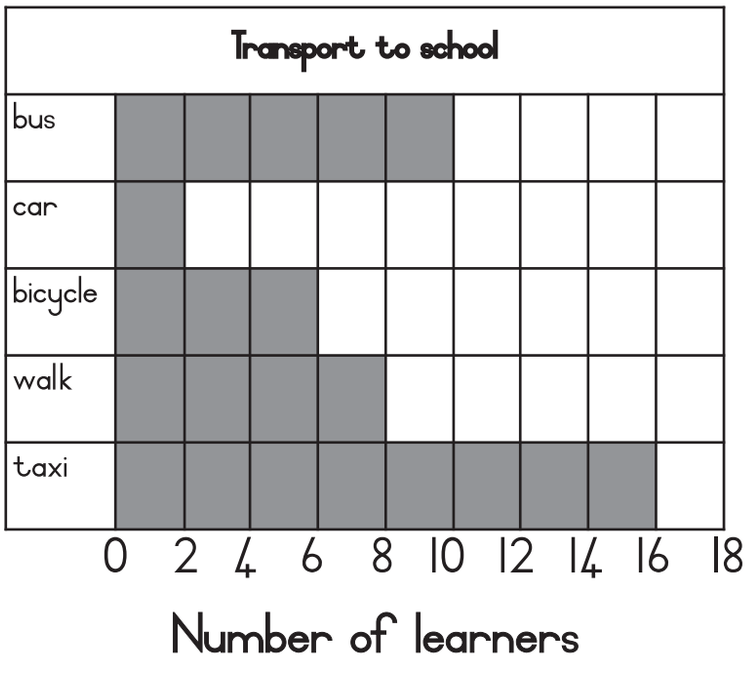


- Ask questions such as:
  - What is the least favourite colour?
  - What is the favourite colour?
  - How many more red cars are there than green cars?
  - Which colours have an equal number of cars? etc.

Teaching data handling by interpreting and analysing data

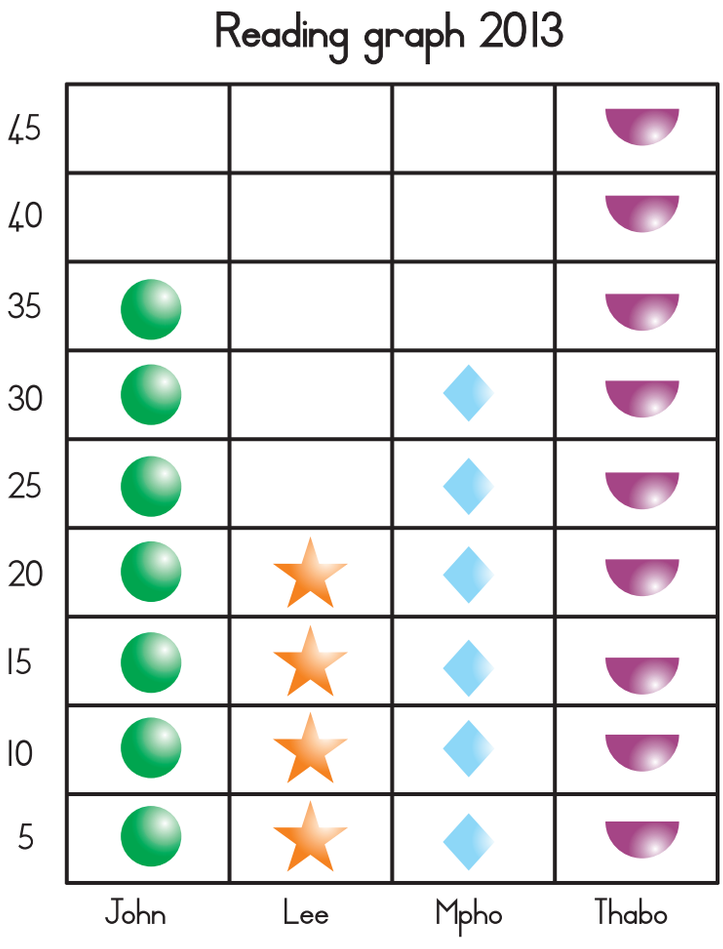
- Learners should be able to interpret and analyse data represented in a list, table, tally table, pictograph and bar graph.
- Learners should be able to answer questions on a graph.
- Learners must be able to interpret a graph correctly by looking at the axis indicating the intervals of numbers on the graph.
- It is important that learners are exposed to the mathematical vocabulary of questions based on graphs.
- Mathematical vocabulary that learners should be familiar with is as follows: Most, least, more than, less than, equal number, the same, altogether, double, half, twice as much.
- If the graph is based on money, mass, length, capacity or time the appropriate vocabulary should be taught and thoroughly explained to learners.
- Here are some examples of graphs and interpretive questions for you to select from and to work through with your learners.
- In each case, look at the graph and answer the questions.

1. Number of learners



- How many learners come to school by taxi?
- Which transport is the least popular?
- Which transport is the most popular?
- What is the difference between the number of learners coming to school by taxi and bicycle?
- How many learners are in the Grade 3 class?

2. Number of books



- Who read the most books in 2013?
- How many books did Mpho and John read altogether?
- How many less books did Lee read than Thabo?
- Who read the least books in 2013?
- What is the difference between the number of books Thabo and John read?

3. Study Lerato's shopping list below and answer the questions that follow:

Shopping list for your birthday:	Quantity
cake	1
candles	9
chips	15
Coke	15
straws	15
sweets	30
chocolates	15
paper plates	15
paper cups	15
balloons	7

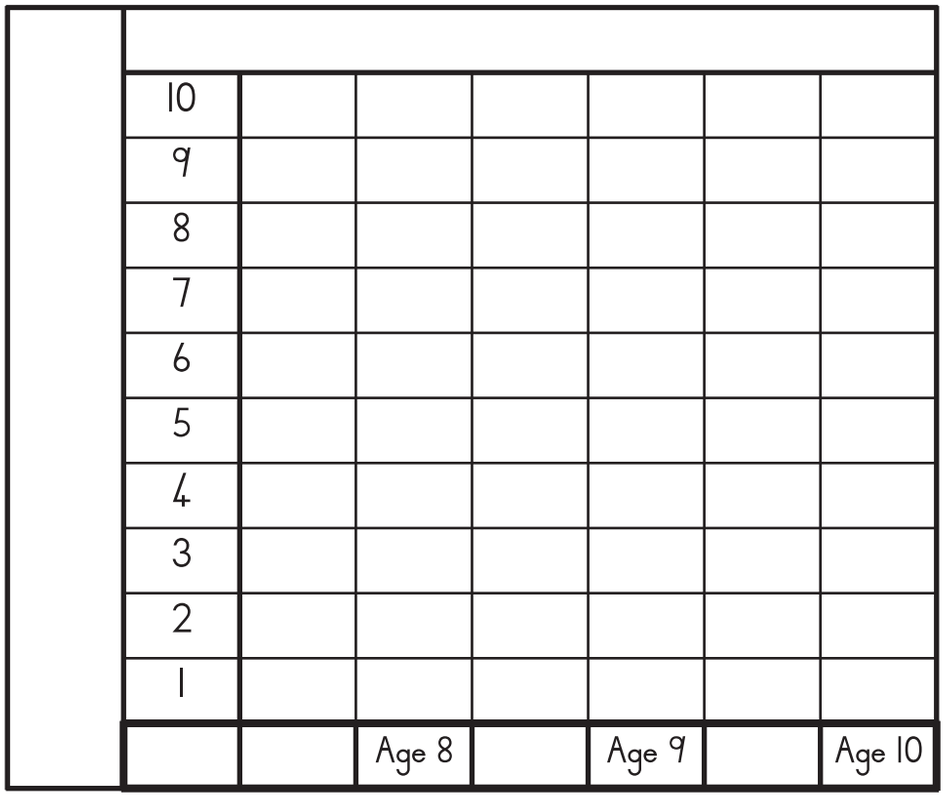
- A balloon costs R3. How much will Lerato pay for her balloons?
- Which item will there be the most of at the party?
- Which item will there be the least of at the party?
- If there are 15 plates at the party, how many children do you think will be invited?

4. Learners should be able to interpret a table and draw a graph based on the data on the table.

- Give the learners a list of 8 learners' names and their respective ages.

Name of friend	Age
Mpho	8
Harry	10
Paul	8
Shirley	9
James	8
Martha	10
Lebo	8
Lalie	8

- Learners should draw a bar graph on the graph template as shown below **(see printables)**.



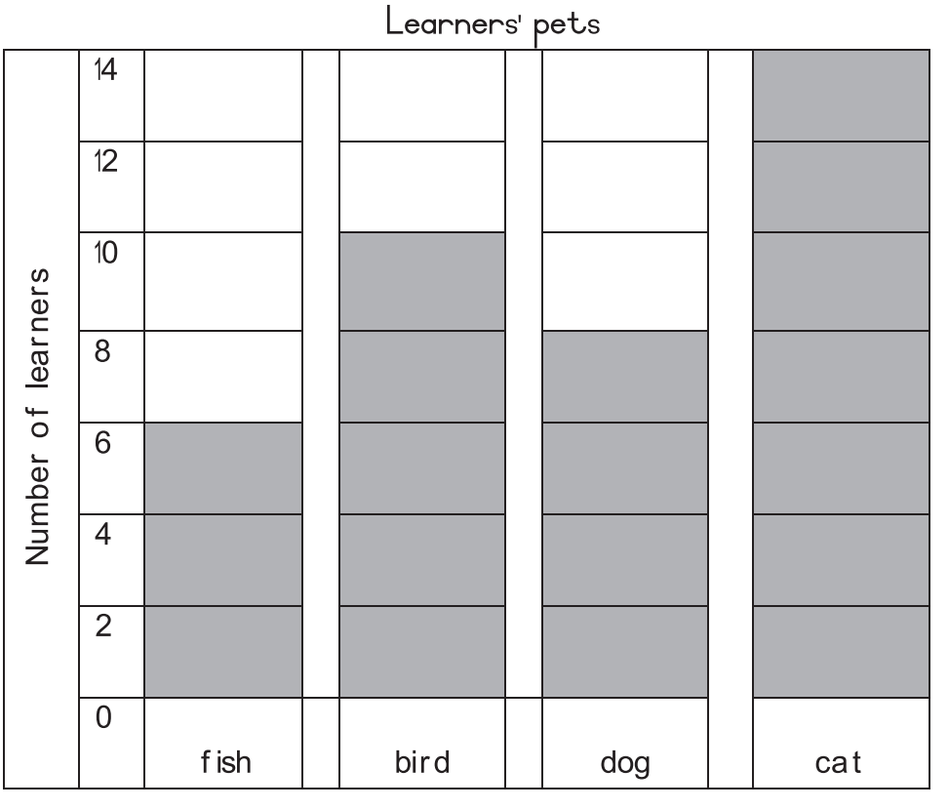
- Learners should label the graph.
- Learners should label the vertical and horizontal axes.
- Learners should complete the bars using counts.

Notes:

Another example of how knowledge of data handling can be tested

ANA 2014 Grade 3 Mathematics Item 23

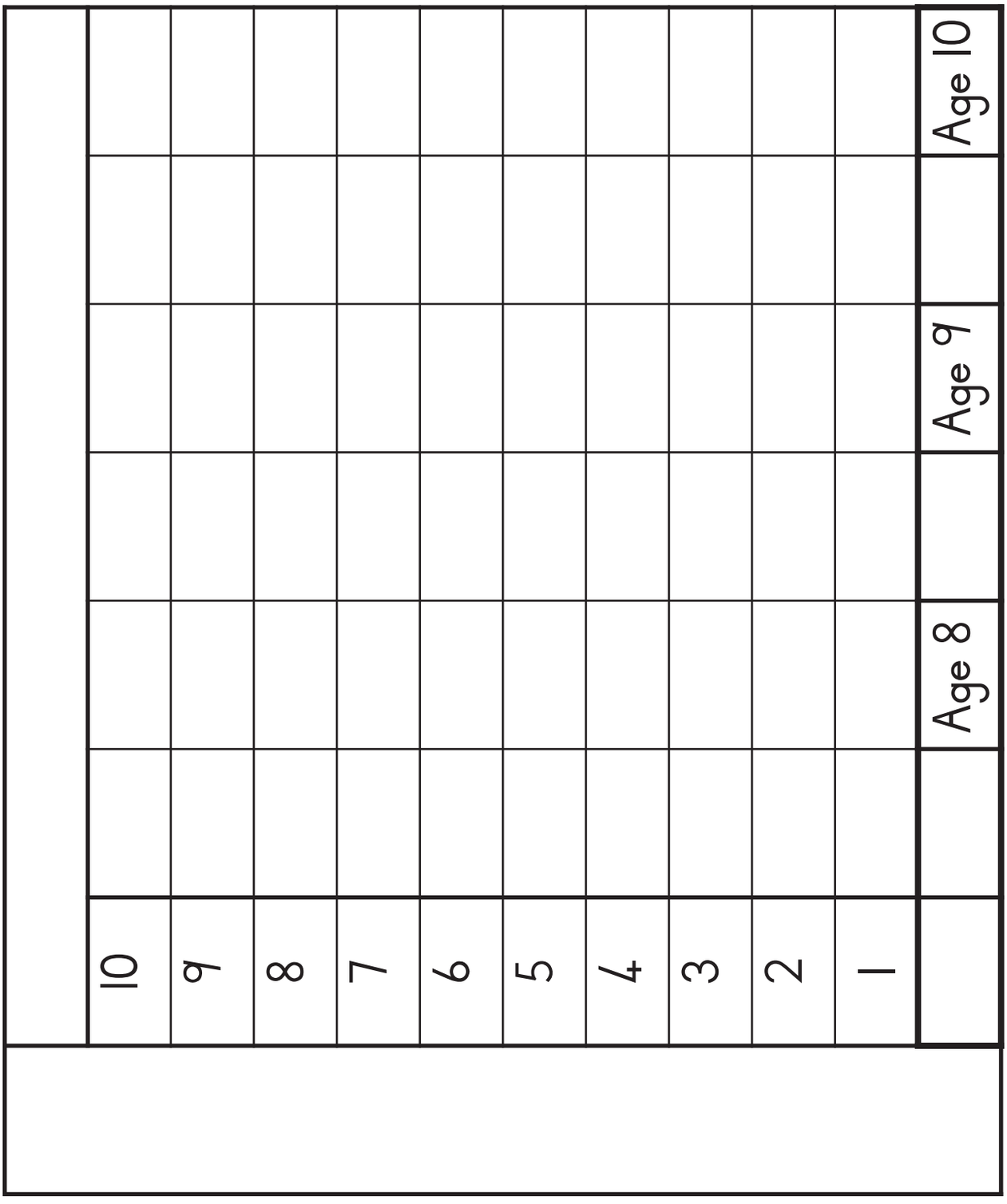
23. Study the bar graph and answer the questions that follow.



23.1 Which is the most popular pet? _____

23.2 How many learners like dogs? _____

Printable: Graph template



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